THE DYNAMICS OF AGRICULTURAL INSURANCE
AND CONSUMPTION SMOOTHING

By

Gerald Gesicho Omae Nyambane

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Economics

2005
ABSTRACT

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The majority of previous studies on agricultural risk management use static models and, for the most part, ignore use of borrowing and lending as an alternative method of managing risk. This study examines the interaction between credit, insurance, and liquidity constraints using a simple dynamic model for a risk averse farmer who uses revenue insurance to manage risk and also borrows and lends subject to a credit constraint. Theoretical and numerical results are provided to support the hypothesis that liquidity constraints can have a large impact on optimal insurance decisions.

Three theoretical results are derived with the following implications. First, with no liquidity constraints, a risk-averse farmer will choose full coverage of actuarially fair insurance, even if borrowing and lending is allowed. Second, with no liquidity constraint a positive premium loading reduces optimal coverage level below full coverage. These two results show that in a dynamic model with no liquidity constraints, insurance choices are not influenced by the desire to smooth consumption, as long as complete and well-functioning credit markets exist that permit efficient consumption smoothing to take place. Third, even if insurance is actuarially fair, a binding liquidity constraint reduces optimal coverage below the full coverage level. Implying that, a binding liquidity constraint may cause farmers to purchase insurance less often than would be expected in the absence of the constraint.
The numerical model was solved for a representative farm from Adair County in Iowa and provides the following implications. First, with complete and well functioning credit markets: (i) the maximum allowable coverage of actuarially fair insurance will always be optimal; (ii) at moderate premium loading (e.g. 30%), the maximum of 85% coverage allowed in practice will still be optimal; and (iii) at relatively high (e.g. 60%) premium loading the maximum allowed coverage will no longer be optimal except for highly indebted farmers. Second, a liquidity constraint causes a reduction of coverage below the maximum allowed level, even for actuarially fair insurance. A binding liquidity constraint limits (or eliminates) the insurer’s ability to borrow for current expenditures including consumption and insurance. This causes him/her to not insure out of current wealth because current consumption is too valuable. Finally, an area-based insurance scheme exposes insurers to residual uninsurable risk which may preclude them from purchasing insurance, even if it is actuarially fair and there is no liquidity constraint. Hence, subsidies may be necessary to encourage the maximum allowable coverage.

This study has two main conclusions: (1) as long as complete credit markets exist and the farmer can borrow and save freely, consumption smoothing has no effect on insurance decisions if insurance is moderately priced and there is no residual uninsurable risk; (2) if residual uninsurable risk and/or a liquidity constraint exist then consumption smoothing can have a significant impact on the optimal insurance decision and, in some cases, self-insurance will be preferred over formal insurance.
DEDICATION

I would like to dedicate this dissertation to my parents Prisca and Samson Nyambane and to my sister Hilda
ACKNOWLEDGMENTS

I would like to thank my dissertation committee, Drs. Steve Hanson (Chair), Robert Myers, Roy Black, Jack Meyer, and Laura Cheney for providing me with valuable insights in their respective areas of expertise. My deepest gratitude and respect go to Dr. Myers who has worked tirelessly and patiently with me on numerous drafts of this dissertation. I will be forever indebted to this wonderful teacher and gentleman.

Special thanks go to Drs. Hanson and Black who provided me with financial and moral support to complete this dissertation. They inspired me and gave me the opportunity to realize my dream of completing a Ph.D. The honesty, kindness, and respect that I received from these individuals is exceptional. Thank you Dr. Hanson. Thank you Dr. Black.

I would also like to extend my sincere thanks to Dr. Richard Bernsten who encouraged me to come to Michigan State University and served as my major professor during my masters degree program. Throughout my life in graduate school, Dr. Bernsten has continued to be my mentor and good friend. In the same vain, my sincere thanks also go to Dr. Thom Jayne who provided me with financial support for my masters studies and supervised my research during that time. Without their support, it would not have been possible to pursue a Ph.D.

I am also indebted to many of my colleagues who have supported me in one way or another. A special mention goes to Kofi Nueve, Lesiba Bopape, Yanyan Liu, Dr. Brady Deaton (University of Guelph), and Dr. Patricia Makepe (University of Botswana).
These individuals provided me with constant encouragement and friendship that I will treasure for the rest of my life.

I am deeply grateful to my family for their unlimited love and support during this long process of completing my studies. My parents and siblings had to make great sacrifices during this period. I am blessed to have them and for that I thank God.
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CHAPTER ONE
INTRODUCTION

1.1 Problem Statement

Farmers have a wide array of instruments for managing income risk. Futures and options contracts, forward contracts, and other derivative pricing instruments have been available for many years. Multiple-peril crop insurance, which triggers payoffs based on individual-farm yield shortfalls, has long been an option to manage yield risk. More recently, area-yield insurance which triggers payoffs based on county yield shortfalls has been made available to many farmers. The latest innovation in risk management is direct protection against revenue shortfalls through revenue insurance. Revenue insurance is currently being offered under a variety of designs including individual farm revenue insurance and area revenue insurance, and alternative methods for valuing yield or revenue shortfalls.

There is a large literature on the optimal use of risk management instruments. The bulk of this literature focuses on pricing instruments, such as futures or forward contracts, and uses static models (e.g. Ederington, 1979; Anderson and Danthine, 1981; Kahl, 1983; Myers and Thompson, 1989). Several authors have recognized the importance of dynamic hedging and made important contributions in developing dynamic models for hedging with pricing instruments (e.g. Anderson and Danthine, 1983; Karp, 1987; Martinez and Zering, 1992; Vukina and Anderson, 1993; Myers and Hanson, 1996). A limited number of studies have focused on including multiple instruments in a
farmer’s risk management portfolio. The vast majority of these studies also use static models (e.g. Coble et al., 2000; Wang et al., 1998; Hennessy et al., 1997; Mahul and Wright, 2000). Atwood et al. (1996) is one of the few studies to examine the use of insurance instruments in a dynamic framework.

In a multi-period setting, a potentially important alternative method to manage risk is available which has, for the most part, not been explicitly included in studies on risk management. It is well known that borrowing and lending can be used to smooth consumption across time (Friedman, 1957; Sargent, 1987). That is, in periods of low income a farmer may borrow to maintain a desired consumption level and in periods of high income the farmer can repay the borrowed funds or lend to store wealth for future consumption. Hence, borrowing and lending can be used to counteract the effects of income variability across time.

In addition, the majority of existing studies on optimal risk management either implicitly or explicitly assume that perfect credit markets exist. This assumption implies that farmers can borrow and lend as much as they want at going interest rates. However, lenders usually place a limit on the amount farmers can borrow. Whenever such a borrowing constraint is binding, it imposes a further constraint on the amount of liquid funds available to the farmer. Consequently, the farmer’s ability to finance consumption, insurance, and production activities becomes limited. This suggests that in the presence of a (binding) liquidity constraint, farmer choices of consumption and insurance coverage levels may differ from those under the assumption of perfect credit markets.
Results from static models of optimal insurance coverage for risk averse individuals with von Neumann-Morgenstern utility functions are well known. Full insurance coverage will be demanded if complete insurance is available at an actuarially fair premium (Arrow, 1963). Situations in which this result may not hold have also been studied, especially for non-farm insurance problems (e.g. Parkin and Wu, 1972; Harris and Raviv, 1978; Holmstrom, 1979; Shavell, 1979). Agricultural insurance studies that have analyzed situations in which full actuarially fair insurance may not be the optimal choice have mainly focused on market failures due to moral hazard and adverse selection, which arise from asymmetric information between insured farmers and insurance agents (e.g. Chambers, 1989; Luo et al., 1994; Rothschild and Stiglitz, 1976). As suggested above, however, the presence of credit markets and a binding liquidity constraint may cause insurance decisions to deviate from full coverage, even at actuarially fair premia. With the exception Gollier (2003), no other study was found that examined insurance demand in a dynamic framework with a liquidity constraint.

This study is based on the conjecture that a better understanding of farmers’ risk management behavior may be reached if: (i) analyses are conducted within a multi-period setting, because this would represent the way farmers make actual decisions more closely than single period models; and, (ii) a liquidity constraint is explicitly included in the analysis to account for imperfections and restrictions in credit markets. Few existing studies have analyzed farmers’ risk management behavior in this context. The purpose of this study, therefore, is to explore farmers’ risk management behavior using a dynamic framework in which credit markets may be imperfect. The results are
expected to be of interest to economists and policy makers who are concerned with the
design and use of risk management instruments.

1.2 Objectives of the Study

This study has two interrelated specific research objectives:

1. To derive optimal revenue insurance choices for a farmer operating in a dynamic
   environment who can also borrow and save subject to a liquidity constraint.

2. To examine the sensitivity of the optimal insurance choices for the farmer in (1)
   under a variety of alternative insurance instrument designs and features.

These objectives are accomplished using a dynamic, time separable expected utility
model. Dynamic programming (DP) is used to study optimal decision rules for
consumption, insurance coverage, and credit choices for an individual farmer. In
addition, the approach and results are illustrated via a numerical example using data from
Adair County in Southwest Iowa.

The focus is on revenue insurance because this reduces the dimensionality of the
dynamic programming model, and because revenue insurance is an important component
of current U.S. agricultural insurance offerings. A conceptual dynamic model is
developed in which an individual farmer, whose objective is to smooth lifetime
consumption, uses credit markets and revenue insurance for risk management. Next, the
model is solved to obtain optimal revenue insurance choices: (i) assuming complete
credit markets (no liquidity constraint); and (ii) assuming incomplete credit markets (with
a liquidity constraint). Finally, the model is analyzed under the preceding two
assumptions with a loading on the insurance premium to cover insurer administrative
costs or allow for insurance subsidies. Closed form solutions are often unobtainable because of the complexity of the model, especially in the case of a liquidity constraint. Consequently, numerical methods are used to analyze optimal insurance coverage choices.

The second objective is to take the model developed under objective 1 and investigate optimal insurance choices under alternative insurance designs. In particular, optimal insurance choices are examined when indemnification is based on an area revenue index rather than an individual farm revenue index. This design is consistent with the way area revenue insurance is sold in practice and allows one to investigate the impact of basis risk (imperfect correlation between farm and area revenues) on the optimal insurance choice.

1.3 Organization of Dissertation

The dissertation is organized into six chapters. Chapter two contains a review of selected literature on consumption smoothing and crop insurance. The basic relationship between consumption smoothing and insurance is developed, then a discussion of yield and revenue insurance is provided and the contribution of this study to the broader crop insurance literature is clarified.

A conceptual dynamic revenue insurance model is developed for an individual crop farmer in Chapter three. Optimal insurance policies are derived analytically which provide a theoretical foundation for the optimal insurance choices analyzed numerically using stochastic dynamic programming. The results are then summarized and discussed.

Chapter four provides details of the stochastic dynamic programming model, description of the data used for calibration, and the solution algorithm. Details of
different experimental designs (alternative insurance instrument designs) are also presented. Model validation is also discussed and the model is solved under various underlying assumptions concerning the existence of liquidity constraints. The results are presented and discussed in Chapter five. Finally, Chapter six concludes by summarizing the major results. The strengths and limitations of the study are also highlighted, along with suggestions for future research.
CHAPTER TWO
LITERATURE REVIEW

2.1. Introduction

This chapter reviews the literature on consumption smoothing and crop insurance. The goal of the review is to enhance understanding of the current literature and to clarify the contribution this study makes to the existing literature. Section 2.2 begins with a review of studies from developing countries where credit and insurance markets are incomplete and imperfect. Various strategies used to smooth consumption in these countries are discussed. Next, similar studies from developed countries are reviewed, where it is assumed that credit and insurance markets are more complete. Section 2.3 reviews studies on crop insurance and suggests a possible relationship between consumption smoothing and insurance decisions. Finally, Section 2.4 provides a synopsis of the chapter.

2.2. Consumption smoothing and revenue risk management

Agricultural revenue risk depends on price and yield risks. When resources are allocated at planting time, farmers have to make risky decisions because harvest prices and yields are uncertain. Price risk exists because of unpredictable shifts in supply and demand for farm inputs and outputs. Yield risk, is due to weather variability, disease and pest incidences as well as other natural phenomena such as floods and droughts. The combined effect of the variability in prices and yields leads to variability in revenue. Other things being equal, risk averse farmers prefer stable revenue streams to ones that vary over time, especially, if the downside swings are large. Hence, they may take
actions that will reduce revenue risk.

Agricultural economists have devoted a tremendous amount of effort to studying and developing various instruments farmers use to manage risk. These include, but are by no means limited to, enterprise diversification, hedging in futures and options markets, storage, and crop yield and revenue insurance. Farmers can also mitigate consequences of unanticipated revenue shocks through credit (borrowing and savings). That is, credit can be used to finance consumption during periods of low income. Alternatively, savings accumulated in prior periods can be divested and used to finance current consumption. In practice, farmers use a combination of several of these strategies at the same time. As a result, these strategies interact with one another and often act as substitutes in some cases and complements in others.

The interaction between consumption smoothing and risk management in general is not a new idea. It is covered extensively in the development literature focusing on credit and insurance markets in developing countries. In these countries, formal insurance and credit markets are underdeveloped or not accessible by many rural households. Faced with revenue risk and a lack of formal markets for managing risk, households in these countries use a variety of coping mechanisms to smooth consumption.

Some studies have focused on testing how well households in these countries are able to smooth consumption while others have examined alternative coping strategies empirically. For example, Paxson (1992) tests for consumption smoothing among Thailand rice farmers based on the permanent income hypothesis (PIH). The PIH posits that if credit markets are complete, then transitory income shocks should be smoothed away through borrowing and saving, and should not affect consumption. Using time series data, she ran a regression with household savings as the dependent variable and
transitory and permanent incomes as explanatory variables, among other variables. According to the PIH, the savings function coefficient on transitory income should be unity and the coefficient on permanent income should be zero. She found that the coefficient on transitory income was not statistically significantly different from one. The coefficient on permanent income, however, was positive and statistically significantly different from zero, contrary to the PIH. However, it was much smaller than the coefficient on transitory income. This led her to conclude that the PIH is only partially supported because a higher fraction of transitory income is saved than of permanent income. In other words, she found evidence that Thailand rice farmers smooth consumption, but not to the extent implied by the PIH.

Townsend (1994) also tests for consumption smoothing among households in India but uses a different approach from Paxson. He uses a 10-year panel data set from three villages in southern India and tests for what he terms “full insurance”. He ran a regression with household grain consumption as the dependent variable and household income as one of the explanatory variables. According to the full insurance model, a household’s consumption should be independent of its income. Therefore, the coefficient on income should be zero. He found this coefficient to be statistically different from zero, although its magnitude was very small. Consequently, he rejected the full insurance hypothesis but argued that there was evidence of partial insurance because of the low magnitude of the income effect. The implication of this result is that households smooth consumption but not to the extent implied by “full insurance”. Other studies which have tested the hypothesis of full insurance in developing countries (e.g., Grimard, 1997) also reject full insurance but find evidence of some consumption smoothing.
There are many studies that examine consumption smoothing strategies in developing countries and these vary in their approaches and focus. Rosenzweig and Wolpin (1993) examine the role of bullock purchases and sales in smoothing consumption among rural households in India. They use a structural household model in which they assume that households have no access to credit and so consumption smoothing can only be achieved via accumulation and depletion of bullock stocks. The assumption of no credit at all seems unrealistic because a number of studies have shown that even in extreme cases of missing credit markets, informal credit institutions emerge to provide some form of credit (e.g., Besley, 1995). The authors acknowledge the restrictive nature of this assumption but argue that, in reality, very few households receive loans for consumption purposes. Their study shows that bullock stocks play an important role in consumption smoothing in India.

Lamb (2003) shows that farmers also use off-farm income to smooth consumption. That is, farmers augment their farm incomes by engaging in off-farm employment during seasons of low labor demand on their farms. Using a two-period model he shows that farmers in southern India use more fertilizer (a purchased input) on their farms as off-farm employment increases. He posits that a positive relationship between fertilizer use and off-farm labor supply implies that the off-farm labor market plays a role in consumption smoothing. The logic is as follows. First, increased fertilizer use can be viewed as an ex-ante risk management strategy, especially if the original fertilizer application levels were sub-optimal. Second, to the extent that off-farm employment is available when on-farm labor demand is low, then it is prudent for farmers to seek this employment in order to smooth unanticipated farm income shocks ex-post. In a similar dimension, Rosenzweig and Stark (1989) also find that rural
households with greater farm income volatility are more likely to have a household member in steady wage employment.

Another strand of the literature focuses on examining informal risk-sharing arrangements. Besley (1995) provides a fairly exhaustive summary of the most prevalent non-market risk sharing and credit provision arrangements addressed in this literature. These include credit cooperatives, informal credit and insurance arrangements, rotating savings and credit associations, and extended family networks.

Credit cooperatives typically secure loans from lending institutions on behalf of their members. Individually, these members would be denied access to credit by the lending institutions for various reasons, e.g., lack of collateral, high risk of default, and so on. The cooperatives are usually organized in a way that diminishes problems of adverse selection and moral hazard. In most countries, cooperatives are formal institutions that are regulated by government laws and, in some cases, can secure loans from their governments when commercial banks are unwilling to lend them.

Rotating savings and credit associations, on the other hand, are typically informal institutions formed by groups of individuals with similar interests. Most often, the members of an association reside in the same locality and know each other well. Members of the association make regular contributions to a common kitty to raise funds for credit provision. Members meet periodically to allocate some funds to one member according to laid down rules. The process goes on until all members have been allocated funds and the cycle continues. Hence, the terms ‘rotating’ credit or ‘merry-go-round’ are used to describe these associations.

It is clear that the topic of consumption smoothing and risk management strategies in countries with imperfect credit and insurance markets has been extensively
covered in the existing literature. These studies provide empirical evidence that households in developing countries use several strategies to smooth consumption, including off-farm employment, use of risk-decreasing inputs, informal credit and insurance arrangements, and livestock holdings. Consumption smoothing and risk management have also been examined in developed countries but with a somewhat different focus.

The studies in developed countries, such as the United States (U.S.), have mainly focused on testing the validity of the PIH and investigating the conditions under which it does not hold. As stated earlier, the PIH posits that if agents have rational expectations and credit markets are perfect, so that agents can borrow and save at going interest rates, then desired consumption is determined by permanent income and is independent of current income. These studies use a variety of estimation methods and data sets to test this hypothesis with a general finding that the PIH is rejected (e.g. Flavin, 1981; Hall and Mishkin, 1982; and Zeldes, 1989).

Flavin uses a time series model of aggregate consumption of nondurable goods to test the PIH. She found that the observed sensitivity of consumption to current income was greater than that warranted by the PIH. She termed this ‘excess sensitivity’ of consumption to current income and this issue became the focus of many subsequent studies. Hall and Mishkin (1982) is one example. They use a seven-year panel data set from the Panel Study of Income Dynamics (PSID) to examine the issue of excess sensitivity of food consumption to fluctuations in current income among US families. They found that 80% of consumption followed the permanent income hypothesis and, by inference, about 20% of consumption is excessively sensitive to current income. Their conclusion was that the strong hypothesis that consumption is determined only by
permanent income could be rejected.

Zeldes (1989) builds on the work of Hall and Mishkin, among others, but addresses a slightly different issue from previous studies. He investigates whether liquidity constraints can explain the rejection of the PIH. In a very general sense, a liquidity constrained household is one that is unable to meet desired levels of consumption due to low liquid net wealth. This situation arises mainly when the household is unable to borrow the necessary amounts to smooth consumption over time. A variety of liquidity constraints due to borrowing restrictions are examined in the literature. Some involve interest rate restrictions while others involve quantity restrictions. Zeldes investigates the effect of quantity restrictions on borrowing by restricting the consumer’s net liquid wealth to be nonnegative in all periods. In this respect, the liquidity constraint investigated by Zeldes is essentially a borrowing constraint. Therefore, he uses the terms “liquidity constraints” and “borrowing constraints” interchangeably as do many other authors. These terms are used interchangeably in this study too.

Zeldes derives testable implications for consumption behavior in the presence of borrowing constraints and tests them using the PSID data set. He split the observations into two groups based on the proportion of financial assets held to total income. The group with the lower assets to income ratio was assumed to be liquidity constrained. He explored several splits of the observations to investigate the sensitivity of the results to different liquid assets to income ratios. The most stringent split consisted of one group (group 1) with a ratio of liquid wealth to income equal to zero while the other group (group 2) had a ratio of at least 0.5. He designed a test for each group based on the Lagrange multiplier for the liquidity constraint. Further, he derived an estimate for the
Lagrange multiplier associated with consumption growth above the amount that would, \textit{ceteris paribus}, be predicted by a model with no constraints. If the liquidity constraint is binding then the estimate should have a positive mean for group 1 observations. For the stringent split he found that the estimate for group 1 indicated that the liquidity constraint caused annual food consumption growth to be 4.3\% higher than it would have been without the constraint. Tests from the other splits were less conclusive. However, he drew the conclusion that in general liquidity constraints influence consumption decisions.

Several other studies have also examined the effect of liquidity constraints on consumption smoothing in the U.S. (e.g., Flavin, 1985; Deaton, 1991; and Chah \textit{et al.}, 1995). These authors use various approaches to investigate whether the observed excess sensitivity of consumption to current income is due to the presence of liquidity constraints. The conclusion by all three authors is that excess sensitivity of consumption to current income can be attributed to liquidity constraints. Almost all of the other studies cover similar issues, differing only in emphasis, data, or methods used. Unlike the development literature, the issue of the interaction between consumption smoothing, liquidity constraints, and insurance is not explored. The studies by Atwood \textit{et al.} (1996) and Gollier (2003), reviewed later on in this chapter, are the only exceptions found.

For crop farmers, transitory income is generated through crop revenues which often fluctuate due to seasonality as well as shifts in supply and demand. Such fluctuations are usually transmitted to consumption although they can, in principle, be smoothed through borrowing and savings. When complete credit markets exist, consumption credit is, \textit{ceteris paribus}, demanded and supplied to individuals who are cash strapped in the face of unanticipated low revenue realizations. Crop insurance can also be used to protect the farmer’s revenue from unanticipated downside shocks.
Obviously, crop revenue can be low due to low prices, low yields, or both. Unfortunately, both prices and non-irrigated yields are uncertain at planting time. It is this source of uncertainty that underlies the demand for insurance. That is, the farmer purchases insurance at planting time in order to receive an indemnity if harvest time revenue falls below a given level. Typically, farmers use both credit and insurance to mitigate consequences of revenue uncertainty. Therefore, one would expect demand for credit and insurance to be related. Moreover, if borrowing constraints exist, in the sense that farmers are unable to obtain the amount of credit demanded at the going interest rate, one would expect this relationship between credit and insurance to be affected. The majority of previous studies on crop insurance implicitly assume the existence of perfect credit markets in which farmers can borrow and save freely at the going interest rate. However, as seen from the literature reviewed above, borrowing constraints are a reality, even in the U.S. where credit markets are often assumed to be perfect and complete. The question raised in this study is: How might the presence of borrowing constraints affect optimal crop insurance decisions? This question has not been addressed in the existing crop insurance literature, which is reviewed below.

### 2.3 Agricultural insurance

Insurance offers farmers an opportunity to buy protection against potentially large yield or revenue losses. Crop yield insurance, for instance, guarantees the farmer’s yield will not fall below a given level by compensating the farmer for any shortfalls, valued at a set price. That is, at planting time, a yield guarantee (usually set at an estimate of average yield) is established, and the farmer is offered yield-coverage options specified as percentages of the yield guarantee. In the case of Multiple Peril Crop Insurance
(MPCI), the yield coverages range from 50% to 85% in 5% increments. Let $Q_g$, $b$, and $Q_h$ represent the yield guarantee, coverage level, and the farmer’s actual yield at harvest time, respectively. Then the farmer’s decision problem is one of selecting the coverage level, $b$, for which he pays a premium in exchange for the opportunity to receive an indemnity payment if his realized yield falls below the insured yield. That is, the farmer receives an indemnity payment if $(bQ_g < Q_h)$ and zero otherwise. In addition to selecting yield coverage, the farmer also selects a price coverage at which the yield loss will be valued. Under the MPCI program, the farmer can select a price coverage of up to 100% of the price set by the Risk Management Agency (RMA). The indemnity is thus given as $p_g \max\{ (bQ_g - Q_h), 0 \}$ where $p_g$ is the price guarantee. Premium rates are usually set by RMA based on the expected indemnity for a given coverage, conditional on information available at planting time.

The concept behind using insurance in risk management is risk pooling. Risk pooling involves combining the risks faced by many individual farmers facing uncorrelated losses who contribute to a common fund via insurance premiums. Any individuals in the pool who experience losses are then compensated for their loss using funds from the common pool. To be successful at risk pooling, insurers must sell policies to many different farmers with uncorrelated (or less than perfectly correlated) risks resulting in a portfolio that is less risky than the individual policies. With uncorrelated risks, the probability of a significant proportion of farmers in the insurance pool having a claim at the same time is very low. If risks are highly correlated across insureds, the common fund will have a significant probability of going bankrupt, perhaps leading to failure of the insurance market (Duncan and Myers, 2000).
Crop insurance, especially under the MPCI program, has been widely studied since the 1980s (Knight and Coble, 1997, provide a detailed survey of this literature). When the Federal Crop Insurance Act of 1980 was passed, the “goal was to create an insurance program that would replace disaster relief measures while operating on an actuarially sound basis with limited government interference” (Goodwin, 1993, p.425). However, participation in crop insurance programs continued to remain modest over the years, giving rise to subsequent legislative changes to the original program (see for example, Goodwin, 1993; Coble et al., 1996; and Knight and Coble, 1997 for details).

Studies have focused on several issues in an attempt to understand and explain the apparent failure of crop insurance programs to perform as expected. Several authors have suggested that this failure is primarily due to problems of moral hazard, adverse selection, and systemic risks. Moral hazard occurs when a farmer buys insurance and then alters production practices to increase the likelihood of receiving an indemnity. Adverse selection arises when potential buyers of insurance are better informed about their potential magnitude of loss and/or probability of loss than the insurer. This leads to purchase of insurance mainly by high risk farmers who have the greatest chance of receiving indemnities. In the long run, this may lead to insurance claims exceeding premium revenue, and hence failure of the insurance market. Finally, systemic risks exist when many of the insured farmers suffer losses at the same time, leading to failure of risk pooling. In essence, moral hazard and adverse selection effects are manifestations of market failure due to asymmetric information, while systemic risk problems tend to be associated with natural catastrophes, such as drought or floods.

Many studies have addressed asymmetric information problems within crop insurance. Examples include Skees and Reed (1986) who examined adverse selection
problems with the Federal Crop Insurance (FCI) program of the mid-nineteen eighties. Using both theoretical and empirical analysis, they showed that the FCI procedure for establishing premium rates and yield guarantees under the Actual Production History (APH) program was vulnerable to adverse selection problems. Further, they showed that yield discounts were required to provide incentives to farmers with higher expected yields who would, in turn, pay lower premia in order to avoid crowding out lower-risk farmers from the insurance market. This finding was consistent with measures already put in place by the FCI to deal with low participation rates. The theoretical underpinning of their finding, and their suggested remedy for adverse selection problems, is now well known following the work of Rothschild and Stiglitz (1976) on the economics of imperfect information. Rothschild and Stiglitz developed an insurance design that offers alternative contracts based on the risk types of those purchasing insurance. Risk premia and coverage would be set such that high-risk farmers and low-risk farmers would voluntarily choose the contracts designed for them. Rothschild and Stiglitz showed that if the insurance markets are competitive, then an equilibrium can be reached under this ‘self-selection’ scheme.

Coble et al. (1997) empirically investigate moral hazard effects on MPCI indemnities using data from wheat farms in Kansas. They found that moral hazard had a significant effect on expected indemnities, but only in years of poor production. In years with favorable growing conditions, there was no effect. To mitigate these problems in crop insurance markets, Miranda (1991) advocated the use of area-yields instead of a farmer’s individual yield to calculate indemnities and premia. Under the area yield

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1 As stated by Miranda, the idea of area-yield insurance design was first proposed by Halcrow (Miranda, 1991 p.234).
scheme, moral hazard is eliminated because the farmer cannot alter the likelihood of receiving an indemnity based on area yields. Adverse selection problems are also reduced because information regarding area yield distributions is more generally available to insurers than information about individual yield. Miranda developed a theoretical framework for the proposed area yield insurance design and applied it to a sample of soybean growers from western Kentucky. He concluded that “... for most producers, area yield insurance would provide better overall yield risk protection than individual yield insurance” (p. 242).

Several studies since then have been devoted to finding optimal as well as operational contract designs for area-yield insurance (e.g. Skees et al., 1997; Mahul, 1999; and Vercammen, 2000). Skees et al. provide a detailed description of the Group Risk Plan (GRP) which is an example of an area yield insurance product offered by the Federal Crop Insurance Corporation (FCIC). They worked in collaboration with FCIC personnel to develop practical methods of setting premium rates, choosing coverage areas, forecasting yields, and so on, for the GRP. They emphasized that a key requirement for the area yield insurance scheme to be successful is the existence of systemic risk. Systemic risk refers to the portion of individual farm risk that is perfectly correlated with the area yield, while nonsystemic risk refers to the component that is uncorrelated with the area yield. It follows then that the higher the systemic risk, the greater the risk reduction that the farmer can get under the area yield insurance contract (Miranda, 1991, Proposition 3).

Mahul (1999) derived a closed form optimal area yield insurance contract using an expected utility model. Further, he applied the model to the sample of soybean farmers used by Miranda (1991) to obtain an “ideal” contract. His model is similar to that used by
Miranda except that Miranda used a mean-variance framework. Mahul’s main finding was that the optimal coverage level equals the ratio of the covariance of the farmer’s yield and area yield to the variance of the area yield. As pointed out by Mahul, this ratio is the well known beta in the Capital Asset Pricing Model (CAPM), and measures the sensitivity of farm yield to movements in area yield. This finding is consistent with the point made by Skees et al. (1997) concerning the importance of systemic risk when implementing the area yield insurance scheme.

Vercammen (2000) extends the analysis of Mahul and derives an optimal area yield insurance contract in which the farmer’s desired area yield trigger level is higher than the maximum value allowed by the insurer. He finds that the optimal contract is such that a lump sum payment be made whenever this constraint is binding, i.e. the indemnification schedule is discontinuous. He recognizes the practical difficulties of implementing such a contract and proposes a non-linear indemnity schedule that mimics the lump sum payment feature of the optimal contract. He uses graphical analysis to illustrate that the proposed non-linear indemnity scheme is more efficient than an optimized standard contract, such as the one developed by Mahul. The magnitude of the efficiency gains or the feasibility of implementing the proposal for using a non-linear indemnity schedule are not addressed in the study.

So far the discussion has focused on area yield insurance only in the context of being a solution to moral hazard and adverse selection problems. In this regard, the importance of having systemic risk for this kind of insurance scheme to succeed has been stressed. However, systemic risks pose serious problems for risk pooling and could be a serious challenge for the insurance market. This is because insurance companies may not be able to diversify systemic risks if relatively large areas are ‘homogenous’ with respect
to systemic risk. A good example of such a problem is when there is a natural disaster such as drought or flood. In such cases, yield losses can be huge and spread over large areas and indemnities may bankrupt the insurance fund. This type of systemic risk resulting from natural disasters is sometimes referred to as ‘catastrophic risk’.

Various studies have looked at the problems catastrophic risks pose for insurance markets (e.g., Miranda and Glauber, 1997; Duncan and Myers, 2000). Miranda and Glauber define a measure for systemic (catastrophic) risk as a ratio of the coefficient of variation of total indemnities paid to the coefficient of variation of total indemnities that would be paid if indemnity payments were independent. Further, they use a simulation model to estimate these ratios for the largest ten U.S. insurers. They found that U.S. crop insurers face portfolio risks that were 22 to 49 times larger than they would be if indemnity payments were independent. Therefore, they concluded that systemic risks faced by crop insurance agencies were substantial compared to those faced by other insurance agencies such as, auto, homeowners, and workers’ compensation, among others. To deal with the problem of catastrophic risks these authors suggest use of reinsurance.

Duncan and Myers (2000) develop an insurance market equilibrium model and use it to show that reinsurance does not, in general, facilitate an equilibrium if the cause of private insurance market failure is catastrophic risk. However, they show that if an equilibrium already exists, then reinsurance will enhance both farmer and insurer participation. Further, they show that a reinsurance subsidy helps facilitate an equilibrium by expanding the opportunity set of available equilibria. The implication of this result is that subsidized reinsurance may be necessary to encourage the establishment of a crop insurance market if it is missing because of catastrophic risks.
The general consensus across many of the studies that have focused on catastrophic risk is that failure of (private) crop insurance markets can be due to systemic risks in addition to asymmetric information problems. Ideally, these problems could be addressed through a reinsurance scheme for primary crop insurers. In practice, however, there has been no substantial development of private crop reinsurance markets. This is perhaps because reinsurance companies are required to hold relatively large financial reserves to cover losses whenever they occur. However, as stated by Duncan and Myers (2000), there are few (tax) incentives for holding such large reserves, and doing so may expose the concerned companies to risk of hostile takeovers. Indeed, as some authors have concluded, private reinsurance by itself may be inadequate to encourage primary crop insurers to cover catastrophic risks (e.g. Skees and Barnett, 1999; and Duncan and Myers, 2000). Therefore, some level of government support, in the form of subsidization in the reinsurance market may be necessary. Other strategies have also been proposed including the use of derivative securities (e.g. catastrophe-linked bonds), options based on catastrophes, and weather derivatives in conjunction with reinsurance. Work in this area is still on-going (see for example Mahul, 2001a, 2001b; Miranda and Vedenov, 2001; and Turvey, 2001).

A recent development in the agricultural insurance market is the availability of a revenue insurance program known as Income Protection. This program was first introduced in 1995 in Iowa and Nebraska and has since been made available to other regions (Hennessy et al., 1997). The generic form of a revenue insurance contract provides indemnities as the \( \max(0, k - y) \) where \( k \) is a constant guaranteed revenue floor and \( y \) is realized revenue (Gray et al., 1995; Hennessy et al., 1997). Thus, revenue insurance guarantees a certain level of revenue. All other things being equal, revenue
insurance can be used for protection from declines in both crop yields and prices. But couldn’t farmers have protected their income from falling below a given level by using a portfolio of futures contracts and existing yield insurance instruments? Some authors have attempted to address this question by analyzing the farmers’ risk management in a portfolio setting.

Coble et al. (2000) use a combination of analytical and numerical techniques to evaluate the relationship between alternative insurance designs and hedging. Revenue insurance is found to lower hedging demand compared to a similar level of yield insurance. This suggests that revenue insurance may indeed be partially replicated by a portfolio of yield insurance with futures contracts. Wang et al. (1998) use a two-period model to explore the impacts of alternative designs of yield insurance contracts on the choice of a risk management portfolio for a representative corn farmer using numerical solution techniques. They found that the performance of individual yield insurance was relatively more sensitive than area yield insurance when considered in a portfolio setting with futures and options. However, they did not explore use of revenue insurance in that study. In a subsequent study, Wang et al. (2000) attempt to answer the question of whether revenue insurance can substitute for price and yield risk management instruments. They explore the use of various portfolios under a variety of design specifications using a combination of numerical simulation and optimization methods to solve their model. Revenue insurance is found to outperform a combination of futures and yield insurance only if replacement pricing is used in the design. When replacement pricing is used, the price guarantee is given as $p_g = \max(p_p, p_h)$ where $p_p$ and $p_h$ are the pre-planting and harvest time futures prices, respectively. Hence, the indemnity schedule under the replacement pricing scheme is given as $\max(p_p, p_h)\max\{(bQ_g - Q_h), 0\}$ where $b$,
$Q_g$ and $Q_h$ represent coverage level, yield guarantee and the farmer’s actual yield at harvest time, respectively. This implies that if the farmer has a yield shortfall and the harvest time futures price exceeds the pre-planting futures price, then replacement pricing will lead to a higher indemnity payment than would be received without it. Wang et al. (2000) found that without replacement pricing yield insurance outperforms revenue insurance regardless of whether futures are used or not.

Clearly, the literature on yield and revenue insurance is limited both in the number of studies that have considered these instruments within a portfolio setting, and in the extent to which available studies have addressed this issue. In particular, all the studies cited above have only looked at the problem within a static framework. Static models implicitly assume that credit markets are complete and therefore, farmers wishing to borrow and save at the going interest rate will be able to do so. However, evidence from the consumption smoothing literature suggests otherwise. Some households face borrowing constraints which in turn constrain their liquidity with implications for optimal consumption. In the presence of liquidity constraints, consumption is highly correlated with current income. Therefore, it is logical to expect that liquidity constraints affect insurance decisions. This interaction between insurance, consumption, and liquidity constraints cannot be studied using static models. It can only be studied within a dynamic framework. However, previous studies have paid little attention to dynamics.

Atwood et al. (1996) is one of the few studies to examine the use of insurance instruments in a dynamic framework. They analyzed the impact of federal price support programs and crop insurance on profitability, financial survival rates, and capital structure for wheat farmers in Montana. Their findings showed that a farmer’s first response to risk is the restriction of how much debt to use. They also showed that
availability of price supports and crop insurance allowed farmers to service higher levels of debt and also that price supports and insurance were themselves substitute risk management instruments. Revenue insurance and hedging decisions were not considered in their study.

Although not directly related to crop insurance, there is an important study by Gollier (2003) which examines insurance demand in a dynamic framework. He develops a dynamic model and uses it to investigate the impact of precautionary savings on consumption insurance demand. He considers the case of a consumer who receives a constant flow of income but faces a 10% chance of losing 75% of that income each year. The consumer can manage the risk of income loss through savings and/or through formal insurance subject to a positive net wealth constraint (liquidity constraint) each period. Gollier solves the model numerically to determine the optimal deductible for the consumer as a function of wealth available at the beginning of the period. He concluded that accumulation of wealth induces consumers to significantly reduce their demand for insurance relative to what classical static insurance models suggest. The implication of this result is that consumers with large wealth may not insure at all.

The current study contributes to this literature by exploring optimal revenue insurance decisions in a dynamic framework when the farmer can also borrow and lend subject to a credit constraint. Based on the well known results from dynamic hedging models, one can surmise that analyzing revenue insurance in a dynamic setting can potentially shed new insights on farmers’ inter-temporal revenue risk management behavior. The approach here differs from that of Gollier (2003) in several respects. In this study the optimal insurance coverage is calculated directly while Gollier (2003) solves for optimal deductible. In addition, the model here is calibrated to the agricultural
insurance market while Gollier (2003) examines insurance demand only in a stylized fashion. Also, the impact of basis risk on insurance demand is examined in this study but not by Gollier (2003).

2.4. Summary

Revenue risk is by far the most important type of risk faced by crop farmers as both yields and prices are uncertain at planting time. Risk averse farmers are concerned with fluctuations in their revenue streams. Faced by the need to smooth their consumption over time, they usually take actions to mitigate the consequences of the risky decisions they have to make.

Historically, crop farmers have used various risk management strategies, such as enterprise diversification, contracting, vertical integration, hedging in futures markets, futures option contracts, storage, and crop yield insurance. More recently, direct revenue insurance has become available to many farmers. The existing literature has dealt with many of the issues related to the use of the traditional instruments in managing price and yield risk.

This study takes explicit account of the joint interaction between farmer insurance decisions and their borrowing and saving decisions. Hence, the first objective of this review was to examine the relationship between consumption smoothing and insurance. The interaction between consumption smoothing and risk management has been extensively covered, especially, in the development literature. Three conclusions emerged from this literature, which was reviewed in the first part of the chapter. First, there is empirical evidence that households in developing countries use several strategies to smooth consumption including off-farm employment, use of risk-decreasing inputs,
informal credit and insurance arrangements, and livestock holdings. Second, because credit and insurance markets are missing, imperfect, or simply inaccessible, farmers in these countries use a variety of non-market risk sharing and credit provision arrangements such as, informal credit, rotating savings and credit associations, and extended family networks. Third, the interaction between consumption smoothing and (formal) crop insurance is not covered in that literature, perhaps because of lack of crop insurance markets in developing countries.

Next, associated literature was examined in a developed country setting where complete credit and insurance markets are usually assumed to exist. There are two main findings. First, there is empirical evidence that in general U.S. households smooth consumption through borrowing and savings. Second, and more important, some households face borrowing constraints which limit their ability to smooth consumption at desired levels. This evidence points to a possible relationship between borrowing and insurance decisions. Since borrowing constraints affect consumption decisions they should also affect crop insurance decisions. This interaction between borrowing constraints, consumption smoothing, and insurance has not been comprehensively addressed in the literature. The purpose of this study is to address this gap in the literature.

The second and final objective of this review was to provide a discussion of yield and revenue insurance so as to clarify the contribution of this study to the broader crop insurance literature. Selected studies on yield and revenue insurance were reviewed covering the conceptual, analytical, and empirical issues of contract design, premium rate setting, and crop insurance market problems. The relevant finding is that both yield and revenue insurance have primarily been studied using static models. Analyzing revenue
insurance in a dynamic setting can potentially shed new insights into farmers’ inter-temporal risk management behavior. Hence, this study will also contribute to existing literature by examining the use of revenue insurance within a dynamic framework while allowing crop farmers to smooth their consumption through savings and borrowing subject to borrowing constraints.
CHAPTER THREE

A CONCEPTUAL DYNAMIC MODEL OF AGRICULTURAL INSURANCE AND CREDIT

3.1 Introduction

In this chapter several needs are addressed. First, a general discussion of the dynamic programming approach used in this study is provided. Next, a conceptual dynamic model for analyzing revenue insurance choice for a risk-averse crop farmer is developed. Specifically, a consumption smoothing model is used to examine the effect of credit and liquidity constraints on optimal insurance choice. Finally, analytical results on optimal insurance choice are provided using the model. These results are derived under varying assumptions concerning the existence of a liquidity constraint and actuarial fairness of the insurance premium. The model provides some interesting theoretical insights as well as providing a foundation for the numerical analysis which follows later in the dissertation. The rest of the chapter is organized as follows. Dynamic programming is described next in Section 3.2. The conceptual model is laid out in Section 3.3. The theoretical results on insurance choice with no liquidity constraint are provided in Section 3.4 and insurance choice with a liquidity constraint in Section 3.5. Lastly, a summary of the chapter is provided in Section 3.6.
3.2 Dynamic programming

In this section a description of the dynamic programming method using a generic problem is provided as a preamble to the specific stochastic dynamic model used in this study. The dynamic programming method was developed by Richard Bellman and others during the 1950s. The main principles can be illustrated as follows. Consider a state of nature being observed over a given time horizon that has been divided into periods. In each period, the state of nature is observed and a decision is made. The decision influences the state to be observed next period and, depending on the current state and decision made, an immediate reward is gained. The expected sum of rewards from the current period to the end of the planning horizon given that optimal decisions are made is represented by a value function. The value function in the current period and the one next period are related through a functional equation often known as Bellman’s equation. Starting from the last period, the functional equation is maximized step-by-step backwards to the current period to get optimal decision rules that depend on the period and the current state of nature. In other words, an optimal policy function is obtained for each period. This method of determining the optimal policy function is based on Bellman’s principle of optimality which states that “an optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision” (p. 83). Thus, as time advances, there is no incentive to depart from the original plan.

A generic dynamic optimization problem can be characterized by Bellman’s equation as follows.
\begin{equation}
V_t(s_t) = \max_{x_t} \left[ f_t(s_t, x_t) + E_t V_{t+1}(s_{t+1}) \right] ; \quad 0 \leq t \leq T
\end{equation}

\begin{equation}
\text{s.t.} \quad s_{t+1} = g_t(s_t, x_t, \epsilon_{t+1})
\end{equation}

Here, \( \{V_t\}_{t=0}^{T+1} \) is a sequence of value functions representing the optimized values of the state at time \( t \); \( s_t \) is a vector of state variables that define the decision environment at time \( t \) but are not under the direct control of the decision maker at that time; \( x_t \) is a vector of decision variables chosen at time \( t \) under the direct control of the decision maker at that time; \( s_{t+1} = g_t(s_t, x_t, \epsilon_{t+1}) \), is a vector of transition equations that link the state and control vectors, and describe the evolution of the state vector through time; \( f_t(s_t, x_t) \) is a vector of return functions and represent the immediate reward in period \( t \), given the state vector \( s_t \) and the control vector \( x_t \); and \( \epsilon_{t+1} \) is a vector of random shocks which introduce uncertainty into the future path of state variables because future realizations of the process are uncertain at time \( t \) when the current control variables must be chosen. The additive form of the functional equation implies that \( V_t(.) \) is a linear function of a sequence of return functions over the time horizon of the optimization problem.

The probability density function of the state variable is usually represented by a discrete-time transition probability matrix that maps the stochastic state variables from one decision period to the next. For illustrative purposes, assume two states of nature \( \{s_1, s_2\} \). The elements in the transition probability matrix, \( \Pi \), mapping the states at \( t \) to the states at \( t+1 \) are given by
(3.2) \[ J = \begin{bmatrix} \pi_{11} & \pi_{21} \\ \pi_{12} & \pi_{22} \end{bmatrix} \]

where \( \pi_{ij} \) gives the probability that state \( i \) at \( t \) will be followed by state \( j \) at \( t+1 \) and therefore the row elements in \( II \) sum to one. As an example, the second element in the first row is defined as:

(3.3) \[ \pi_{21} = \text{Prob}(s_{t+1} = s_2 | s_t = s_1). \]

For finite horizon problems, the optimal policy function is found by iterating on Bellman’s equation starting at the terminal period and moving backwards recursively through time. In practice, a computer algorithm is often used for this purpose.

However, when the horizon is infinite, i.e., \( T \rightarrow \infty \), one cannot proceed with the backward iteration algorithm to solve the problem because there is no last period in which to start. The dynamic programming problem, conditional on the initial conditions, is the same at each point in time since there is always an infinite number of periods left to go. If the discount factor for future rewards is less than one and, if the return and transition equations are time invariant, then the value function will also be time invariant and is denoted as \( V(s_t) \) with the time subscript dropped. Solutions to this class of infinite horizon problems will also be time-invariant policy functions. In practice, a solution algorithm for the infinite horizon problem is based on one of two basic methods: value function iteration or policy function iteration (Kennedy, 1986; Miranda and Fackler, 2002). Iteration based on these methods entails starting with an initial guess of the value function (which can be a vector of zeros) or policy function, respectively, and then iterating on Bellman’s equation repeatedly, using updated value or policy functions, until some convergence criterion is reached.
For some non-stochastic problems, closed form solutions can be found using analytical techniques. However, closed form solutions are more difficult to find for many stochastic dynamic problems. Therefore, numerical dynamic programming (DP) algorithms have to be used. The main advantage of numerical DP techniques is that they provide flexibility which permits the resolution of inter-temporal optimization problems even when the functions embedded in Bellman’s equation are not continuous and differentiable, or the underlying variables are stochastic. Unfortunately, because the recursive problem must be solved for all possible values of the state variables, the main disadvantage of the numerical approach is that the size of the model increases exponentially as the number of state and decision variables increases. This is what Bellman termed the curse of dimensionality and is often cited as the reason for the low adoption of this apparently appealing stochastic optimization technique.

In the section that follows, focus is on developing the conceptual model used in this study. In addition, analytical results on optimal insurance choice are provided under a variety of assumptions, leaving numerical analysis to be the focus of later chapters.

3.3 Conceptual Dynamic Model

The interaction between credit, insurance, and liquidity constraints is explored using a simple dynamic model. Consider a farmer who lives for an infinite number of periods (or cares about his/her heirs) and maximizes expected utility of lifetime consumption assuming a standard, discounted, time additive utility specification:\(^2\)

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t)$$

where \(E_t\) is expectation conditional on information available at the

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\(^2\) The assumption of infinite life is made for convenience only. Virtually identical results can be obtained if a finite lifetime is assumed, either with or without a bequest motive.
beginning of period $t$; $\beta$ is a discount factor representing the farmer’s rate of time preference in consumption; $U(.)$ is a concave, differentiable, and strictly increasing utility function; and $c_t$ is period $t$ consumption.

At the end of each period (harvest time), the farmer receives realized crop revenue, $y_t$, which, as of the beginning of the period (planting time), is an identically and independently distributed (iid) random variable with a cumulative distribution function $F(\xi) = \text{Prob}[ y_t \leq \xi ]$ defined over the support $[a, b]$ so that $F(a) = 0$ and $F(b) = 1$. The farmer can take out insurance at the beginning of each period to insure against revenue shortfalls which, may occur at the end of the period. To accomplish this, the farmer chooses a coverage level, $k_t$, and pays a premium, $P(k_t)$, at the beginning of the period in order to receive an indemnity, $\max(0, k_t - y_t)$, which is not paid until the end of the period, when crop revenue is realized. The current period’s consumption, $c_t$, is also chosen at the beginning of the period. Also, at the beginning of the period, the farmer incurs production costs, $x_t$, which are known with certainty at that time. Next period’s production costs, however, are uncertain and are assumed to be iid with a cumulative distribution function $Z(\zeta) = \text{Prob}[ x_{t+1} \leq \zeta ]$ defined over the support $[c, d]$ so that $Z(c) = 0$ and $Z(d) = 1$. All production costs, consumption, and insurance premium expenditures are paid out of wealth, $w_t$, available at the beginning of the period. Any wealth not used to finance consumption, production costs, or the insurance premium can be invested at the risk free interest rate, $r$.

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3 The iid assumption can be generalized to a Markov process without changing the main results and implications, but this generalization complicates the theoretical exposition greatly, and also increases the state space making numerical solution more difficult.
Intuitively, wealth in the model represents the farmer’s aggregate net worth while the risk free interest rate is a weighted average return on any wealth invested. Hence, the model assumes that there is only one store of wealth and that is (implicitly) a risk-free bond that can be traded long or short (i.e. the farmer can borrow or lend at the risk-free rate) with no other productive assets. Further, the model considers the farmer’s insurance problem on a per acre basis and therefore, there is no provision for growth through purchase of more land. In reality, a farmer can have, say, 500 acres and choose to insure only a fraction of this acreage. Such flexibility is not provided for in the model. The model is a simplified representation of the farmer’s problem which can be used to gain insights on the joint interaction between insurance and borrowing and saving decisions. Also, it is assumed that the farmer does not buy any other productive assets such as corporate bonds, or livestock, etc., because these are not necessary for studying the issues investigated in this study. Finally, it is assumed that there are no other government programs and that all revenue is insurable with no uninsurable risk.

With these assumptions, the transition equation for wealth is given by:

\[ w_{t+1} = (1 + r)[w_t - x_t - c_t - P(k_t)] + \max(k_t, y_t) . \]

To this point no constraint has been imposed on \( w_t \) so that borrowing (\( w_t < 0 \)) as well as lending (\( w_t > 0 \)) are both allowed. In some cases, a liquidity constraint is imposed:

\[ w_t - x_t - c_t - P(k_t) \geq m \]

where \( m \) is the minimum net wealth position that is allowed at any period, \( t \). Clearly, this constraint restricts the amount of borrowing that can be undertaken to finance current consumption, production costs, and insurance premium payments. Deaton (1991) has argued that such liquidity constraints are a pervasive feature of reality in both developing and developed economies, and can go a long way towards explaining observed
correlations between income and consumption, which are much higher than would be predicted by models without liquidity constraints (Pissarides, 1978; Flavin, 1985; Zeldes, 1989; Jappelli, 1990; Chah et al., 1995). If such liquidity constraints exist because creditors are unwilling to finance current consumption due to unknown or uncertain future repayment ability, then a risky activity such as agriculture is likely to face even higher liquidity constraints than the rest of the economy.

The farmer’s optimal choice of insurance will depend on the nature of the premium schedule, \( P(k_t) \). Here, the premium schedule is assumed to be given by:

\[
P(k_t) = (1 + \alpha)(1 + r)^{-1} E \max(0, k_t - y_t)
\]

(3.6)

where \( \alpha \) is the premium loading factor and \( E \) is expectation conditional on information available at the beginning of the period\(^4\). If \( \alpha = 0 \), then the premium is actuarially fair because it equals the discounted expected indemnity payment. If \( \alpha > 0 \), then there is an additional loading factor to cover insurer costs and provide insurer profit. If \( \alpha < 0 \), then the insurance is being subsidized. Changes in the loading factor, \( \alpha \), will be examined to investigate how this alters the incentive to insure.\(^5\)

The farmer’s problem is to choose a set of contingency plans for consumption and

\(^4\) In some of the crop insurance literature the premium loading factor is referred to as a ‘wedge’ on the premium.

\(^5\) Because the purpose of this study is to examine the role of consumption smoothing and liquidity constraints on insurance choice, rather than informational asymmetries per se, no explicit allowances are made for moral hazard and adverse selection in the insurance premium schedule (3.6). Note, however, that the equilibrium effects of moral hazard and adverse selection could be included implicitly in the loading factor, \( \alpha \). That is, it could be argued that the equilibrium effect of moral hazard and adverse selection might just be to raise \( \alpha \) above what it would otherwise be for this particular farmer.
insurance coverage that satisfy Bellman’s functional equation:

\[
(3.7) \quad v(w_t) = \max_{c_t,k_t} \left\{ U(c_t) + \beta E_t v(w_{t+1}) \right\}
\]
as well as the transition equation (3.4), the liquidity constraint (3.5), the insurance
premium schedule (3.6), and the transversality condition,

\[
(3.8) \quad \lim_{t \to \infty} \beta^t w_t = 0 \quad \text{which rules out perpetual borrowing.}
\]

Sufficient conditions for solving this problem are satisfied immediately by the
concavity of the utility function, \( U(.) \). The first order conditions for solving Bellman’s
equation (3.7) subject to (3.4) and (3.5) are:

\[
(3.9) \quad U'(c_t) - \beta(1 + r)E_t U'(c_{t+1}) - \lambda_t = 0 \quad \text{and}
\]

\[
(3.10) \quad G(k_t) = 0 ;
\]

where \( \lambda \) is the Lagrange multiplier on the liquidity constraint,

\[
G(k_t) = -(1 + \alpha)F(k_t)[\lambda_t (1 + r)^{-1} + \beta E_t U'(c_{t+1})] + \beta \int k_t U'(c_{t+1})dF(y_t) \quad \text{and the}
\]

fact that \( v'(w_t) = U'(c_t) \) (from the envelope theorem) has been used as well as

\[
P'(k_t) = (1 + \alpha)(1 + r)^{-1} F(k_t) \quad \text{[from differentiating the premium function (3.6)].}
\]

These first order conditions are the necessary conditions which characterize a solution to
the problem and are used in the proofs below. In general, solutions to such problems take
the form of a set of contingency plans \( c_t = c(w_t) \) and \( k_t = k(w_t) \) which, together with an
initial value for wealth \( w_0 \), the transition equation (3.4), and a set of realizations for the
random crop revenues \( \{ y_t \}_{t=0}^{\infty} \), and production costs \( \{ x_t \}_{t=1}^{\infty} \), determine the entire future
path of consumption, insurance, and wealth (Sargent, 1987; Deaton, 1991).

Next, the effects of consumption smoothing and liquidity constraints are studied
by examining optimal insurance choice in the dynamic model, both with and without
liquidity constraints.
3.4 Insurance Choice with No Liquidity Constraint

With no liquidity constraints one might expect the farmer to use the credit market to smooth consumption and the insurance market to manage revenue risk. This intuition is confirmed in the following proposition.

**Proposition 1:** With no liquidity constraint, then a farmer faced with actuarially fair insurance ($\alpha=0$) will choose full insurance coverage ($k_t = b$).

**Proof:** With no liquidity constraint, then $\lambda_t=0$ and with actuarially fair insurance, $\alpha=0$. The proposed solution for this case is full insurance, $k_t = b$. With full insurance, then realized revenue plus indemnities are guaranteed to sum to $b$ every period, so there is no remaining risk. With these substitutions, $G(k_t)$ in (3.10) becomes:

\[
G(k_t) = -\beta F(k_t)E_tU'(c_{t+1}) + \beta \int_0^{k_t} U'(c_{t+1})dF(y_t).
\]

But because $c_{t+1}$ is non-stochastic under full insurance, then $E_tU'(c_{t+1})=U'(c_{t+1})$ and $\int_0^{k_t} U'(c_{t+1})dF(y_t) = U'(c_{t+1})F(k_t)$.

With these substitutions, $G(k_t)=0$ is immediate, which shows that full insurance is optimal for any strictly concave utility function under the conditions of the proposition.

This is the familiar result from static insurance theory that risk averse agents facing actuarially fair premiums will take full insurance coverage (Arrow, 1963; Hofflander et al., 1971; Doherty, 1975). Here it is shown that this result also holds in a dynamic model with consumption smoothing, as long as the insured can lend and borrow freely at the same interest rate.

If the insurance is not actuarially fair, one might expect increases in the loading
factor, \( \alpha \), to lead to lower coverage levels. This intuition is also confirmed in the following proposition.

**Proposition 2:** With no liquidity constraint, an increase in the loading factor, \( \alpha \), above zero will reduce the optimal insurance coverage level below \( b \), so that full coverage is no longer optimal.

**Proof:** With no liquidity constraint, then \( \lambda_t = 0 \) and with a positive loading factor, \( \alpha > 0 \). Evaluating the derivative function \( G(k_t) \) in (3.10) under these conditions and, at the full coverage outcome, \( k_t = b \), gives:

\[
G(k_t) = -(1 + \alpha) \beta F(k_t) E_t U'(c_{t+1}) + \beta \int_0^1 U'(c_{t+1}) dF(y_t).
\]

But because \( c_{t+1} \) is non-stochastic at the full coverage outcome, then

\[
E_t U'(c_{t+1}) = U'(c_{t+1}) \quad \text{and} \quad \int_0^1 U'(c_{t+1}) dF(y_t) = U'(c_{t+1}) F(k_t).
\]

Making these substitutions in (3.12) gives:

\[
G(k_t) = -\alpha \beta F(k_t) U'(c_{t+1}) < 0.
\]

Because this derivative is unambiguously negative for all consumption choices, and the utility function is strictly concave, \( k_t \) must be reduced below the full coverage level in order to satisfy the first order conditions and make the choice of coverage level optimal. Therefore, as the loading factor, \( \alpha \), is raised above the actuarially fair level, \( \alpha = 0 \), then the optimal coverage level falls below full coverage, \( b \).

This is consistent with another standard result from static insurance theory that optimal coverage decreases with increases in the loading factor (Hofflander et al., 1971; Doherty, 1975). Here, it is shown that a similar result holds in a dynamic model with
consumption smoothing and no liquidity constraint.

These results have shown (not surprisingly) that in a dynamic model with no liquidity constraints, insurance choices are not influenced by the desire to smooth consumption, as long as complete and well-functioning credit markets exist that permit efficient consumption smoothing to take place.

3.5 Insurance Choice with a Liquidity Constraint

With a liquidity constraint one might expect the farmer to use insurance to help smooth consumption, as well as to manage risk. This intuition is confirmed in the following proposition.

Proposition 3: With a binding liquidity constraint, a farmer faced with actuarially fair insurance \((\alpha=0)\) will choose less than full insurance coverage \((k_t < b)\).

Proof: With a binding liquidity constraint, then \(\lambda_t > 0 \forall t\) and with actuarially fair insurance, \(\alpha = 0\). Evaluating the derivative \(G(k_t)\) in (3.10) under these conditions and at the full coverage outcome, \(k_t = b\), gives:

\[
G(k_t) = -F(k_t)[\lambda_t(1+r)^{-1}] + \beta E_t U'(c_{t+1})] + \beta \int k_t U'(c_{t+1})dF(y_t).
\]

But because \(c_{t+1}\) is non-stochastic at the full coverage outcome, then \(G(k_t)\) can be written as:

\[
G(k_t) = -F(k_t)\lambda_t(1+r)^{-1} < 0.
\]

Because this derivative is unambiguously negative for all consumption choices, and the utility function is strictly concave, \(k_t\) must be reduced below the full coverage level in order to satisfy the first order conditions and make the choice of coverage level optimal. Therefore, even with
actuarially fair insurance, the existence of a binding liquidity constraint will cause farmers to reduce their coverage below the full insurance level.

This is the main analytical result of the study. It shows that if farmers are faced with a liquidity constraint, then even if insurance is actuarially fair, they may choose reduced or even zero coverage, depending on the severity of the constraint and their attitudes towards risk. With a binding liquidity constraint, the opportunity cost of insurance, measured in terms of current consumption to be forgone, become too high causing insurance coverage to be reduced.

3.6 Summary

A conceptual dynamic model is developed for studying the revenue insurance behavior of a crop farmer whose objective is to maximize the expected utility of lifetime consumption. The effects of consumption smoothing and liquidity constraints are investigated by examining optimal insurance choice under a variety of assumptions concerning the insurance premium schedule. Three theoretical results are obtained and can be summarized as follows. First, with no liquidity constraint, a risk-averse farmer will choose full coverage of actuarially fair insurance. Second, with no liquidity constraint a positive loading on the insurance premium reduces the optimal coverage level below full coverage. Third, even if insurance is actuarially fair, a binding liquidity constraint reduces the optimal level coverage below full coverage. This is the main analytical finding of this study and can be, at least, a partial explanation for why there may be weak participation in crop insurance programs.
CHAPTER FOUR
STOCHASTIC DYNAMIC PROGRAMMING MODEL

4.1 Introduction

This chapter describes the numerical stochastic dynamic programming model used in this study. The data used to calibrate the numerical model are described together with the other parameters of the model. The solution algorithm and model validation procedures are also discussed. Finally, several experimental designs for model solutions are discussed. These characterize the diverse environments under which revenue insurance is offered to farmers as well as the typical constraints they face.

The numerical model provides explicit optimal insurance decision rules under a variety of assumptions, which could not have been obtained from the theoretical model. In particular, the impact of a liquidity constraint, different levels of premium loading, basis risk, and the farmer’s initial level of wealth on insurance choice are all examined. These results are discussed in Chapter 5. In this chapter focus is on specifying the variables and parameters of the model.

4.2 The Stochastic Dynamic Programming Model

Closed form solutions to stochastic DP models usually cannot be obtained due to the complex nature of the models. One method of obtaining explicit results to the farmer’s optimization problem is via discrete time numerical programming techniques (Miranda and Fackler, 2002), which are discussed in this chapter. For convenience
Bellman’s functional equation of the optimization problem is re-stated below:

\[
(4.1) \quad v(w_t) = \max_{c_t,k_t} \left\{ U(c_t) + \beta E_t v(w_{t+1}) \right\}
\]

subject to

\[
(4.2) \quad w_{t+1} = (1 + r)[w_t - x_t - c_t - P(k_t)] + \max(k_t, y_t)
\]

\[
(4.3) \quad w_t - x_t - c_t - P(k_t) \geq m
\]

\[
(4.4) \quad P(k_t) = (1 + \alpha)(1 + r)^{-1} E_t \max(0, k_t - y_t)
\]

\[
(4.5) \quad \lim_{t \to \infty} \beta^t w_t = 0
\]

\[
(4.6) \quad \tilde{y}_t \sim f(y_t | e_t)
\]

\[
(4.7) \quad \tilde{x}_{t+1} \sim g(x_{t+1} | \eta_{t+1})
\]

\[w_0 \text{ and } x_0 \text{ given.}\]

As defined earlier, the variables \(w_t, x_t, c_t, \) and \(y_t\) represent wealth, production costs, consumption, and crop revenue, respectively; \(v(.)\) is the current value function, \(P(k)\) is the insurance premium schedule where, \(k_t\) is the level of insurance coverage (guaranteed revenue level); \(\alpha\) is the premium loading factor; \(r\) is the constant interest rate; \(m\) is the minimum net wealth position allowed under a liquidity constraint; \(U(.)\) is a strictly increasing, differentiable and concave utility function; \(\beta\) is a discount factor; and \(E_t\) is the expectations operator conditional on information available at \(t\). Current period
crop revenue, $y_t$, and next period’s production costs, $x_{t+1}$, are random variables at the beginning of period $t$ with probability density functions $f$ and $g$, conditioned on random shocks $\varepsilon_t$ and $\eta_{t+1}$, respectively. Note that $y_t$, which represents realized crop revenue at the end of the period (harvest time), is a random variable because it is unknown as of the beginning of the period (planting time).

To solve the model the Adaptive Stochastic Dynamic Programming (ASDP) software developed by Lubow (1994, 1995, 1997, 1999) was used. The ASDP algorithm is coded in C++ and is fully compatible with the Microsoft visual C++ compiler, which was used. Even though the ASDP algorithm was developed primarily for solving wildlife and fisheries management problems, it provides a generalized, flexible, efficient, and user-friendly means to define and solve a wide range of stochastic dynamic programming problems. Other dynamic programming algorithms which can be used to solve such problems include the GPDP (Kennedy, 1996) and the DDPSOLVE (Miranda and Fackler, 2002) algorithms, among others. The latter two algorithms are coded in BASIC and MATLAB, respectively. The ASDP was preferred over other algorithms for two main reasons. First, it has undergone extensive debugging and input validation and, as a result, has become an efficient dynamic programming algorithm with a user-friendly interface. Second, and more important, it has several built-in features that greatly reduce
computation time and computer memory requirements. Further discussion of the ASDP algorithm is deferred until Section 4.4. This is done to facilitate a clearer description of how the algorithm is implemented with the data used in this study. Therefore, the data is described first before providing details of the ASDP algorithm.

4.3 Model Calibration and Data

Conceptually, all the state and control variables in the model are continuous except for insurance coverage which is discrete because farmers can only purchase coverage in pre-determined proportions of insurable revenue. For convenience, however, in the dynamic programming model used here a discrete approximation of the continuous state and control spaces is used. Discrete approximation is used for two main reasons. First, the ASDP algorithm is designed to handle problems with only discrete state and control variables. Second, the algorithms that can handle problems with continuous states and continuous controls generally require the value function to have well defined first and second derivatives with respect to all its arguments (Miranda and Fackler, 2002). Due to the insurance instrument in the model used here, this differentiability requirement may not be met.

---

6 Using ASDP we were able to solve a problem with a discrete state and control variable space size of 35,376 x 2,178 using a Pentium III processor with only 13 GB hard disk space and 128 MB RAM. On the other hand, the maximum problem size that we could solve in MATLAB using the DDPSOLVE algorithm was 1,050 X 105 and, this was only possible when we used a Pentium IV processor with 40 GB hard disk space and 640 MB RAM.
This study examines two insurance contract designs which are discussed later in this chapter. The first design is an individual (farm) revenue insurance scheme while the second is an area-based insurance scheme. In the former scheme, indemnity payments are triggered by farm revenue whereas in the latter, they are triggered by county-level revenue. Therefore, both farm-level and county-level revenues as well as production costs are required as state variables of the model. Each of these variables is discussed below, beginning with production costs. Further, because the numerical model is based on a representative corn farmer in Adair County, South West Iowa (SWIA), the data used pertains to that region.

4.3.1 Production Costs

Per acre production costs data were obtained from the Economic Research Service (ERS) database on commodity costs and returns for the period 1975 to 2000 (ERS/USDA, 2002). These data are available at the US Corn Belt region level which covers the state of Iowa. These are reported only in nominal values and therefore, they needed to be converted to their 2001 dollar equivalents before they could be used for estimation. To do so, Consumer Price Index (CPI) conversion factors which are available for each year (Sahr, 2002)\textsuperscript{7} were used.

\textsuperscript{7} The CPI conversion factors were obtained from Sahr (2002) who derived them using data from McCusker, J.J. “How Much is That in Real Money?” \textit{Proceedings of the American Antiquarian Society}, 2001, Table A.1 (cited in Sahr, 2002). Sahr’s derivations were based on a 2001 CPI of 177.1.
The CPI measures the average change in prices paid by consumers over time for a ‘typical’ consumption basket of goods and services. In the model, the price of the farmer’s composite consumption good has been normalized to unity in terms of 2001 dollars. Therefore, the net revenue used to finance consumption also needed to be in 2001 dollars to be consistent with the implied price of the consumption good. This means that gross revenue and production costs also needed to be deflated because these are the two components that constitute net revenue. To convert dollars of a given year to 2001 dollars the dollar amount from that year was divided by the respective conversion factor. Table 1 shows the summary statistics for the nominal and real production costs data. These statistics show that, between 1975 and 2000, real production costs ranged from $158 to $267 per acre, with an average of $202 and a standard deviation of $36.

Table 1. Summary statistics for nominal and real production costs ($/acre) for corn farmers in Adair county for the period 1975 - 2000.

<table>
<thead>
<tr>
<th>Costs</th>
<th>No. of Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>26</td>
<td>126.66</td>
<td>24.44</td>
<td>75.84</td>
<td>161.07</td>
</tr>
<tr>
<td>Real</td>
<td>26</td>
<td>202.38</td>
<td>35.66</td>
<td>157.61</td>
<td>266.72</td>
</tr>
</tbody>
</table>

Time series econometric procedures were used to get an empirical distribution for production costs. First, the graph of production costs was examined to investigate possible trends or patterns. As can be seen from the graph in Figure 1, the nominal costs data series appears to have an upward trend which, reverses to a downward trend when these costs are converted to 2001 dollars. No other systematic patterns are observed.
Next, the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the real production cost series were examined for evidence of autocorrelation. The ACF and PACF plots provide insights as to whether production costs contain an autoregressive structure or not. In general, most time series data tend to have autoregressive structures because current and past values are often serially autocorrelated. If the data series is autoregressive then a model that explicitly takes into account the nature of the autocorrelation is required. In addition, the ACF and PACF
plots also provide insights on the nature of the autocorrelation present. Furthermore, they can be used to infer whether the series is stationary or not. If a series is non-stationary (has a unit root) the ACF is persistent. That is, shocks to the process have permanent effects and, standard distribution theory breaks down and the usual hypothesis testing inferences can no longer be made.

* A priori the search for an empirical distribution was restricted only to stationary processes to facilitate use of the numerical algorithm. This implies that the ‘best’ distribution among a class of alternative stationary distributions was chosen. Hence, distributions with a time trend were not considered. This restriction is justified here because this study is based on a theoretical model and is not a practical decision tool. Nonetheless, an explicit attempt is made to mimic, as much as possible, an empirical setting in order to get a handle on the potential magnitudes of insurance effects.

An examination of the ACF and PACF plots for the production costs showed that the ACF declined geometrically while the PACF went to zero after the first lag. This suggests the series is stationary because there is no persistence in the ACF. A formal test for the presence of a unit root using the Phillips-Perron test also confirmed that there was no evidence of a unit root in the series. The ACF and PACF plots also suggest a low order AR process. However, a formal test for autocorrelation is required. The Ljung-Box Q-statistic test was used for this purpose and evidence of first order autocorrelation was found (p-value = 0.000). This suggests that the data generating process (DGP) for real production costs is a low order AR process. Based on this conclusion, alternative DGPs were hypothesized and investigated. The following is a summary of the procedure used
for the selected DGP. The other candidate models were investigated following similar procedures.

Letting $X_t$ represent period $t$ real production costs $X_t$ was regressed on $X_{t-1}$ and a constant. This implies that the hypothesized DGP for production costs is an AR(1) process with a constant mean. The results of this regression are reported in Table 2 below.

Table 2. Regression results of real production costs on its lagged value and a constant for corn farmers in Adair County for the years 1975 - 2000.

<table>
<thead>
<tr>
<th>No. of obs</th>
<th>dependent variable</th>
<th>constant</th>
<th>parameter on $X_{t-1}$</th>
<th>$R^2$</th>
<th>$F$</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$X_t$</td>
<td>15.3369</td>
<td>0.9049</td>
<td>0.9056</td>
<td>220.5293</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(12.6023)</td>
<td>(0.0609)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2359</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Figures in parenthesis are standard errors and below them are t-test p-values

These results show that the coefficient of $X_{t-1}$ is statistically significant (p-value=0.0000) but the constant term is not significantly different from zero (p-value=0.2359). This would suggest that production costs have a zero long-run mean. Intuitively, however, one would expect production costs to always be nonnegative so the constant term is left in the model. However, it remains important to make sure that there is no residual autocorrelation left in the error term. In other words, one needs to establish if all the predictable variability in the production cost series is explained by past values of
the series.

An examination of the correlogram of the residuals showed that the ACF and PACF cut off immediately. This suggests that the residuals are now essentially white noise. That is, there is no serial autocorrelation in the residuals. Further, a formal test of first-order autocorrelation using the Ljung-Box Q-statistic confirmed that there was no evidence of serial autocorrelation in the residuals at the 5% level (AC coefficient = 0.315, p-value = 0.095). Therefore, the model presented in Table 2 was selected as a reasonable DGP for real production costs.

Following the above results, the transition equation for next period’s production costs was specified as

\[(4.8) \quad X_{t+1} = 15.3369 + 0.9049X_t + \eta_{t+1}\]

where \(\eta_{t+1}\) are iid random shocks with mean zero, standard deviation of 10.3988, minimum value of -20.6641, and maximum value of 21.7230.\(^8\)

The transition equation (4.8) implies that the stochastic nature of production costs is characterized by the random shock, \(\eta_{t+1}\). Therefore, a discrete space for \(\eta_{t+1}\) with corresponding discrete probabilities is required to account for the randomness in production costs. The residuals from the regression model in Table 2 were used to define this (discrete) space as follows.

---

\(^8\) These figures are the descriptive statistics of the residuals from the regression model in Table 2.
First, the residuals were tested for normality using the D’Agostino-Pearson $K^2$ (K-squared) statistic (D’Agostino et al. 1990). Under the assumption of normality, the standardized residuals should have zero skewness and kurtosis of 3. The K-squared test presents one normality test based on skewness and another based on kurtosis and then combines the two tests into an overall statistic. The computed values for skewness and kurtosis for the standardized residuals were -0.171 and 2.646, respectively. Using the K-squared test the null hypotheses that skewness=0 or that kurtosis=3 or that jointly skewness=0 and kurtosis=3 could not be rejected because the p-values were 0.678, 0.967 and 0.917, respectively, for these tests. This suggests that the residuals may be taken to be normally distributed. A caveat about tests for normality, in general, is that they have low power of rejecting the null hypothesis (of normality) in samples with a small number of observations. In spite of this caveat, this study assumed the residuals to be normally distributed as suggested by the test results.

Finally, a discrete distribution for the residuals was specified by dividing the normal density, $N(0,10.3988^2)$, into 15 intervals with corresponding probabilities for the mid-point of each interval. Taking each mid-point of an interval to represent a discrete state in the $\eta_{t+1}$ space, this (discrete) space was specified as a vector of possible random shocks ranging from -$20.66 to $21.34 per acre with $3 increments and a corresponding vector of discrete probabilities (see Appendix Figure A4a).

The discrete distribution, the transition equation (4.8) and the period zero production costs, $X_{0s}$, would completely characterize the discrete production costs space and its corresponding stochastic environment. For the starting value of production costs,
the expected long run mean implied by the regression results was used, i.e. $X_0 = \$160$
per acre. Essentially any other starting value for production costs will give the same
results but it causes the model to take longer to converge. Starting with the long run mean
facilitates faster convergence. Section 4.4 provides details on how the transition equation,
the starting value for production costs, and the discrete random shocks and corresponding
probabilities were programmed in ASDP. This completes the specification of the discrete
production costs space together with its corresponding stochastic environment. The
county-level revenue distribution is described next.

4.3.2 County Revenue Distribution

As stated earlier, both county-level and farm-level revenue distributions are used
in the model. These are discussed in turn, beginning with county-level revenue. Because
data on direct observation of crop revenues is seldom collected, such data are not readily
available. Therefore, the revenue series was estimated as the product of price and crop
yield data.

County-level yields for Adair County were available for the years 1975 to 2000.
These yields were multiplied by the SWIA cash prices to obtain county-level revenue for
each year. The cash price data used were Thursday prices for cash markets in the
Southwestern Crop Reporting District of Iowa.\footnote{I am grateful to the Agricultural Extension Service in the Department of Economics at Iowa State University for providing me with these data.} These were averaged for the month of
October to obtain a figure that was considered to be the annual cash price at harvest time.
Similar to production costs, the revenue for each year was converted to 2001 dollars using the CPI conversion factors discussed above. Table 3 shows the summary statistics for county-level real revenue. These statistics show that, between 1975 and 2000, real revenues ranged from $210 to $686 per acre, with an average of $378 and a standard deviation of $131.

Table 3. Summary statistics for county-level real revenue ($/acre) for corn farmers in Adair county for the period 1975 - 2000.

<table>
<thead>
<tr>
<th></th>
<th>No. of Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Revenue</td>
<td>26</td>
<td>378.33</td>
<td>130.69</td>
<td>210.23</td>
<td>686.32</td>
</tr>
</tbody>
</table>

_A priori_, it is difficult to form reasonable expectations of what the revenue distribution would look like. Therefore, the identification process began by examining the revenue graph looking for possible trends or patterns. As can be seen in Figure 2, real revenue appears to have a downward trend but no other systematic patterns are evident. This trend can, in part, be attributed to the conversion of the nominal series to 2001 dollars.
Figure 2. Graph of gross real revenue ($/acre) for corn farmers in Adair county, South West Iowa.

Next, the sample autocorrelation functions (ACF) and partial autocorrelation functions (PACF) were examined for evidence of serial autocorrelation in the revenue series. The ACF fell from approximately 0.4 to 0.3 after the first lag and remained at that level until lag 5, after which it cut off. The PACF, on the other hand, cut off after the first lag. This suggests that the revenue series is possibly a low order AR process. A formal test for first order serial autocorrelation using the Ljung-Box Q-statistic found evidence of first order serial autocorrelation in the series with an AC coefficient of 0.421 and p-value of 0.023. In addition, the series was also tested for the presence of a unit root using the Phillips-Perron test. The null hypothesis of a unit root was rejected implying that the series is stationary and therefore, the preceding inferences are valid. Similar to production
costs, alternative models for the real revenue data generating process were hypothesized and investigated. The selected model was investigated as follows.

Letting $y_t$ represent period $t$ revenue, $y_t$ was regressed on $y_{t-1}$ and a constant. This implies that the hypothesized DGP is an AR(1) process with a constant mean. The estimated parameters were found to be statistically significant with p-values of 0.024 and 0.015, respectively. In addition, there was no evidence of first order serial autocorrelation (AC coefficient = -0.212, p-value = 0.260). This suggests that real revenue follows a first order autoregressive process. The regression results of this model are reported in Table 4.

### Table 4.

<table>
<thead>
<tr>
<th>No. of obs</th>
<th>dependent variable</th>
<th>constant parameter on $y_{t-1}$</th>
<th>$R^2$</th>
<th>F</th>
<th>Prob&gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$y_t$</td>
<td>198.5919</td>
<td>0.4514</td>
<td>0.2027</td>
<td>5.8467</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(75.6034)</td>
<td>(0.1867)</td>
<td>0.0151</td>
<td>0.0239</td>
</tr>
</tbody>
</table>

Note: Figures in parenthesis are standard errors and below them are t-test p-values.

Based on these results, the transition equation for next period’s (harvest time) county-level revenue was specified as

$$ (4.9) \quad y_t = 198.5919 + 0.4514y_{t-1} + \epsilon_t $$

56
where $\varepsilon_t$ are iid random shocks with mean zero, standard deviation of 115.6841, minimum value of -270.3092, and maximum value of 248.1534. The transition equation (4.9) implies that the stochastic nature of real revenue is characterized by the random shock, $\varepsilon_t$. Hence, a discrete space for $\varepsilon_t$ with corresponding discrete probabilities needs to be defined. The residuals from the regression model in Table 4 were used to define this space as described in the following steps.

First, the residuals were tested for normality using the K-squared test. The null hypotheses that skewness=0 or that kurtosis=3 or that jointly skewness=0 and kurtosis=3 could not be rejected because the p-values for these tests were 0.627, 0.303 and 0.495, respectively. This suggests that the residuals may be taken to be normally distributed.

Next, a discrete distribution for the residuals was developed by dividing the continuous normal density, $N(0,115.6841^2)$, into 15 intervals with corresponding probabilities for the mid-point of each interval. Taking each mid-point of an interval to represent a discrete state in the $\varepsilon_t$ space, this (discrete) space was specified as a vector of possible random shocks ranging from -$270.31$ to $247.98$ per acre, in $37$ increments and a corresponding vector of discrete probabilities (see Appendix Figure A4a). This probability model, the transition equation (4.9) and the period zero gross revenue, $y_{0r}$, completely characterizes the discrete revenue space and its corresponding stochastic environment (see Section 4.4 for programming details in ASDP). The starting value for the county-level revenue, $y_{0c}$, was taken to be the expected long run mean of $362$ per

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10 These figures are the descriptive statistics of the residuals from the regression model in Table 4.
acre. Again, using the long run mean as a starting value is only for numerical convenience in order to facilitate faster convergence. This completes the discrete state space specification for county-level revenue. The farm revenue distribution is described next.

4.3.3 Farm Revenue Distribution

Due to limited farm-level yield data, the farm-level revenue distribution was estimated by adjusting the county-level values to reflect what revenue would be at the farm-level. The assumption here is that farm-level revenues would generally be related to county-level revenue because farmers in a given county face similar technology, weather conditions, and prices. Consequently, the relationship between county-level and farm-level revenue was specified as:

\[
y_{it}^f = \alpha_0 + \alpha_1 y_{it}^c + u_{it},
\]

where \( t \) is a time subscript, \( i \) is an individual farm subscript, \( \alpha_0 \) and \( \alpha_1 \) are parameters to be estimated using the available farm and county data, \( u_{it} \) is the error term and superscripts \( f \) and \( c \) represent farm and county, respectively.

The available farm-level corn yield data consisted of eight years of Actual Production History (APH) data for farms in Adair county from 1985 to 1992. The farm yields were multiplied by the SWIA cash prices to obtain individual farm corn revenue for each of the eight years. These figures were then converted to 2001 dollars using the respective CPI conversion factors. These data were available for 93 farms from the county and therefore, the data set is a panel of annual farm revenues that spans eight years on 93
farms for 744 total observations. To organize the data for estimation, they were first sorted by farm number and then by year. Therefore, the first eight rows represented data for farm one for years 1985 through 1992, the next eight rows represented data for farm two, and so on. Note, of course, that the county revenue for a given year was the same for all farms. With the data thus organized, equation (4.10) was estimated using pooled OLS and the results are presented in Table 5.

Table 5. Pooled OLS results for farm-level revenue on county-level revenue for Adair County

<table>
<thead>
<tr>
<th>No. of farms</th>
<th>dependent Variable</th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$R^2$</th>
<th>$F$</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>93</td>
<td>$y_{it}$</td>
<td>-35.9214</td>
<td>1.0734</td>
<td>0.3911</td>
<td>1189.41</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: Figures in parenthesis are standard errors

Next, the residuals from this regression were tested for normality using the K-squared test and the null hypotheses that jointly skewness=0 and kurtosis=3 was rejected (p-value = 0.000). The computed skewness and kurtosis were -0.49 and 4, respectively. This suggests that the residuals are non-normal, implying that the estimated coefficients are no longer best linear unbiased estimators. When errors are non-normal, the estimators are no longer efficient and in small samples, the standard errors will be biased. Consequently, they cannot be relied upon for hypothesis testing. The estimated coefficients, however, are still consistent and that is what is most important for the
purposes of this study. That is, they give consistent estimates of the parameters for adjusting county-level revenue to farm-level revenue.

Based on these results, the transition equation for next period’s (harvest time) farm-level revenue was specified as

\[(4.11) \quad y_t^f = -35.9214 + 1.0734 y_t^c + u_t;\]

where \(y_t^f\) is the estimated harvest time farm-level revenue given a realization of county-level revenue, \(y_t^c\), which is obtained according to equation (4.9) above, and \(u_t\) are iid random shocks with mean zero, standard deviation 74.1979, minimum value of -302.2889 and maximum value of 257.3683.\(^\text{11}\) The transition equation (4.11) implies that the stochastic nature of farm-level revenue is characterized by the random shock, \(u_t\), in addition to \(\varepsilon_t\) (which acts through \(y_t^c\), see equation 4.9 above). Therefore, a discrete space for \(u_t\) with corresponding discrete probabilities needs to be defined.

Similar to production costs and county revenue, the residuals from the regression model in Table 5 were used to define this space. As stated above, the K-squared test for normality suggests that these residuals are non-normal. Therefore, non-parametric distributions for the residuals were investigated. Specifically, a non-parametric density was fitted to the residuals and compared to the corresponding normal density (i.e. assuming the residuals were normally distributed). In particular, the Epanechnikov kernel was used to estimate a kernel density for the residuals (e.g. Wand and Jones, 1995). Figure

\(^{11}\) These figures are the descriptive statistics of the residuals from the regression model in Table 5.
3 shows the kernel density with an overlaid normal density for comparison. The kernel density estimate suggests that the empirical distribution is somewhat similar to the corresponding normal density although it appears to be narrower and located more to the left than the normal density. The figure suggests that the normal distribution may in fact be a reasonable approximation of the residuals’ distribution despite what the K-squared results suggested.

To help decide which distribution to use, first, the model was estimated assuming that the residuals are normally distributed to obtain the optimal insurance decision rules under this assumption. Second, the model was re-estimated assuming that the probability density function for the residuals is given by the kernel density. Finally, the results obtained under these alternative assumptions concerning the distribution of the residuals were compared. The results were found to be identical. This suggests that either distribution may be used. The results reported in this study are based on the assumption of normality of the residuals. This decision was based on the fact that both production costs and county revenue were found to be normally distributed and, therefore, also assuming farm revenue to be normally distributed maintains consistency on distributional assumptions made without loss of accuracy.
To obtain the discrete state space, the residuals were first fitted to a normal density, $N(0, 74.1979^2)$, to obtain a continuous distribution. Next, a discrete distribution was developed by dividing the continuous normal density into 12 intervals with corresponding probabilities for the mid-point of each interval. Taking each mid-point of an interval to represent a discrete state in the $u_t$ space, this (discrete) space was specified as a vector of possible random shocks ranging from -$264.98 to $257.36 per acre, in $37.31 increments with a corresponding vector of discrete probabilities (see Appendix Figure A4b). This probability model together with the transition equation (4.11),
completes the discrete state space specification for farm-level revenue. Note, of course, that the discrete state specification for county-level revenue is part of this (farm-level revenue) state space via equation 4.11. Note also that the starting value for farm-level revenue does not need to be explicitly specified once the starting value for county-level revenue has been specified. That is, farm-level revenue is treated as an endogenous state variable (see Section 4.4 for programming details in ASDP).

To this point this chapter has described the specific stochastic dynamic model used in this study. It has also discussed the way in which the discrete state space for three of the model’s state variables were specified. Furthermore, because these variables are stochastic, it also described how their respective random environments (random shocks) were approximated with specific discrete shocks and probabilities. The next two sections focus on the other variables and parameters of the model as well as the form of the utility function assumed in this study.

4.3.4 Wealth, Consumption, Insurance Coverage and Premium Specification

In the model, wealth is accumulated according to transition equation 4.2. This means that the amount of wealth the farmer starts with each period depends on the preceding period’s production costs, consumption, net savings, insurance premium, and realized crop revenue as well as any insurance payment received. As discussed earlier in Chapter three, wealth in the model represents the farmer’s aggregate net worth. Further, there is only one store of wealth and that is (implicitly) a bond that can be traded long or short (i.e. the farmer can borrow or lend at the same rate). Furthermore, there is only one
aggregate consumption good. Also, all revenue is insurable and there are no other government programs. Consequently, the discrete state space for wealth should reflect reasonable figures of what the farmer can save, per acre, given his crop revenue and expenses. Under these assumptions, the wealth space was specified as a vector of possible wealth states ranging from -$1,000 to $2,000 per acre, in $20 increments. Each wealth state is taken to be a mid-point of the continuous wealth interval. These figures provide a very wide range of initial wealth levels that span reasonable per acre wealth levels that farmers can have. For example, a farmer with 100 acres and an initial wealth of $100 per acre would have a total of $10,000 of initial wealth. By the same reasoning, the consumption space was specified as a vector of possible consumption choice levels ranging from $0 to $2000 per acre, in $20 increments. Similarly, each consumption level is taken as the mid-point of the continuous consumption interval.

The discrete coverage space for individual farm insurance was specified as a vector of possible insurance coverage levels ranging from 0 to 0.85 in 0.05 increments\textsuperscript{12}. A coverage level of 0 means that the farmer does not insure while a level of 0.85 means that the farmer insures 85% of expected revenue. Thus, a coverage level of 0.85 is the upper limit imposed by the revenue insurance design and, therefore it represents the maximum allowable insurance coverage. The rationale for placing an upper limit on the insurance design is that the deductible implied by the upper limit will act as an instrument

\textsuperscript{12} This specification pertains to the individual revenue insurance design only. Under the area-based scheme, up to 90% coverage can be taken and the insured acres can be scaled by a factor of up to 1.5. Further details on the area-based scheme are provided later in this chapter.
to mitigate problems of moral hazard. Plus this is how the programs actually work in practice.

The insurance premium discrete state space was specified as follows. First, the guaranteed revenue level, \( k_t \), was computed as the long-run mean of expected farm-level revenue\(^{13}\). Given a county-level revenue long-run mean of $362 per acre, the transition equation for farm-level revenue (4.11), and the distribution of \( u_t \), discussed above, \( k_t \) was computed to be $353 per acre. That is, \( y_t^c = $362 \) was substituted in equation 4.11 and 12 possible farm-level realizations were computed, one for each value of \( u_t \). Next, each value was multiplied by the corresponding probability of \( u_t \) and then these products were summed to obtain the expected farm-level revenue. Finally, the insurance premium was computed based on a \( k_t = $353 \) per acre. To illustrate how this was done, let \( y_t^f \) represent each of the 12 possible farm revenue realizations and \( p_i \) represent the corresponding probability for each realization, where \( i=1,\ldots,12 \) is an index of the elements in the corresponding vector. In addition, let \( \text{coverage}_j \) be an element in the insurance coverage space, i.e. \( \text{coverage} = (0, 0.05, \ldots, 0.85) \) implying that \( j=1,\ldots,18 \). The expected payout for a given coverage level, \( \text{coverage}_j \), was computed as

\[
(4.12) \quad \sum_{i=1}^{12} p_i \max(0, k_t \text{coverage}_j - y_t^f )
\]

\(^{13}\) Note that in the theoretical model presented in Chapter 3 the guaranteed revenue level is the maximum possible revenue state. In practice, however, it is computed as the expected farm revenue as done here.
with $k_t$ set to $353$ per acre. In the estimation algorithm, equation 4.12 was programmed so that a premium is computed for each possible coverage level before being used in the premium schedule in equation (4.4).

As for the insurance premium loading factor, $\alpha$, several values were used. Starting with $\alpha=0$ (no loading on the premium) as a base case representing an actuarially fair premium, $\alpha$ was set to -10%, -20%, 30%, and 60% to represent scenarios in which there is a 10% and 20% subsidy as well as a 30% and 60% positive loading on the premium, respectively.

Note that the way the insurance coverage space has been specified above differs from the way it was specified in the theoretical model presented in Chapter Three. First, insurance coverage has been restricted to a maximum of 85% of insurable revenue while in the theoretical model, the farmer could insure up to 100% of the insurable revenue. Second, the revenue guarantee is the expected revenue while in the theoretical model it is the maximum revenue that the farmer can get. That is, according to the theoretical model, the farmer’s revenue is indemnified whenever realized revenue is less than his/her potential maximum revenue. In the calibration above, however, indemnification only occurs when the realized revenue is less than some proportion of expected revenue. As stated earlier, the insurance coverage space was specified this way to mimic, as much as possible, the way crop insurance is offered in practice.
4.3.5 Discount Rate and Utility Function

This study assumes that the farmer’s measure of time preference is constant and is given by $\beta = 1/(1+\delta)$ where $\delta$ is the discount rate. The discount rate reflects the risk free rate of return, $r$, plus a risk premium required by the farmer based on the risk from the stream of cash flows from his/her farm. This study uses a discount rate of 11.25% based on an annual risk free rate of return of 5.43% on a 30-year T-bill (Federal Reserve Board, 2004) and, a risk premium of 5.82% based on Hanson’s (1999) estimate of the historical risk premium for Iowa farmland. The assumption of a constant rate is made for computational convenience. In reality, one would expect the discount rate to fluctuate over time. Taking account of a time varying risk free rate and risk premium is difficult to implement in an infinite horizon model as the one used in this study. To gauge the consequences of the constant discount rate assumption, sensitivity analysis is conducted to gain insights on how the main results would change when the discount rate takes on different values.

The farmer’s utility function was specified as $U(C_t) = C_t^{(1-\gamma)/(1-\gamma)}$ where $\gamma$ is the constant coefficient of relative risk aversion. This utility function exhibits decreasing absolute risk aversion (DARA) which is a desirable property because farmers have generally been found to have DARA preferences (Chavas and Holt, 1990; Saha et al., 1994). The specification above requires explicit assumptions concerning the value of the farmer’s coefficient of relative risk aversion. However, in the existing literature there is limited empirical evidence on what the value is (Meyer and Meyer, forthcoming), even though some studies have found it to be typically above one (Love and Buccola, 1991).
This study sets $\gamma = 2$ based on what previous studies have used (e.g. Wang et al., 2002) and uses sensitivity analysis to investigate the effect of different levels relative risk aversion on optimal decisions.

This completes the discussion on model calibration and data. The next two sections are devoted to how the model was actually operationalized using the ASDP algorithm and then on model validation.

4.4 Model Implementation using the ASDP Algorithm

The objective of this section is to provide a detailed description of how the model was specified and solved using the ASDP algorithm. Here, all the discrete state and control variables and model parameters discussed in the preceding sections are put together. Also, an explanation of how the ASDP utilizes them to provide a solution to the optimization problem is given.

Use of the ASDP algorithm requires four files as inputs. A *state dynamics function* file that computes next period’s state variables given current state variables, current controls and an outcome of the random variables; a *stage return function* file which computes the immediate reward associated with the state and control variables, and outcomes of the random variables in each period; a *terminal value function* file that computes the final reward at the end of the period being modeled; and a *scenario file* which specifies the state and control variables, the distributions of the random variables, the discrete state and control variable spaces, and options for execution and for producing
output. The state dynamics, stage return and terminal value function files are computer programs coded in C++ while the scenario file is not a program and is not coded in C++. Rather, it is written in what the author terms “scenario definition language” that is specific to ASDP but is easy to follow. The scenario file contains data to be used by the program files and is linked to these files via an interface that is in-built in the ASDP program.

The terminal value function in this model is very simple due to the form of the transversality condition (equation 4.5). This function (Appendix, Figure A1) assigns a value zero to the predefined output variable (“result”) and returns this value. That is, the limit of the discounted value of wealth as the time horizon approaches infinity is essentially zero.

The goal of the state dynamics function is to compute the value of each state variable next period \(n_{xt\_stage[j]}\). This computation involves use of various combinations of predefined input variables which are specified in the scenario file. In this study, the state dynamics function (Appendix, Figure A2) uses as inputs current period wealth \(\text{cur\_state[0]}\), current period production costs \(\text{cur\_state[1]}\), and current period county-level revenue \(\text{cur\_state[2]}\); decisions for insurance coverage \(\text{dec[0]}\) and consumption \(\text{dec[1]}\); and outcomes of the random shocks for county-level revenue \(\text{outcome[0]}\), production costs \(\text{outcome[1]}\), and farm-level revenue \(\text{outcome[2]}\). In the top part of the program, the model parameters are declared and, the farm-level revenue \(\text{farm}\) and insurance premium \(\text{prem}\) variables are initialized because they are not predefined inputs to the state dynamics function. Next, two vectors are specified. The first vector \(\text{gross[]}\) contains possible harvest time farm revenue realizations given a period zero county-level...
revenue of $362 per acre. These were computed using equation (4.11) as explained earlier. The other vector (prob[]) contains the corresponding probabilities of the revenues in gross[] being realized.

Given an insurance coverage decision (dec[0]), the vectors gross[] and prob[], the state dynamics function computes an insurance premium that is later used in the equation for next period’s wealth (nxt_state[0]). This computation is based on equation (4.12) discussed earlier. The code for this computation is shown in the middle part of the program. The bottom part of the program shows the equations used for computing next period’s wealth (nxt_state[0]), production costs (nxt_state[1]), farm-level revenue (farm), and county-level revenue (nxt_state[2]). Note that these equations correspond to the transition equations for liquid wealth (4.2), production costs (4.8), farm-level revenue (4.11), and county-level revenue (4.9), respectively. As stated earlier, farm-level revenue is treated as an endogenous state variable. It is computed within the program and used in the computation for next period’s wealth but is not returned by this function as one of the outputs in the nxt_state[] vector. Treating it as an exogenous variable yields exactly the same results but has the disadvantage of increasing the size of the state space.

The goal of the stage return function (Appendix, Figure A3) is to compute current period utility of consumption. The first part of this function is similar to the state dynamics function except for the risk aversion parameter (risk) and the minimum net wealth variable (m). This variable (m) is used when imposing the liquidity constraint (equation 4.3). The function uses as inputs, the consumption decision (dec[1]), current period wealth (cur_state[0]), current period production costs (cur_state[1]), and the
insurance premium (*prem*) which is computed as in the state dynamics function. The bottom part of the program provides instructions for computing the utility of consumption and assign this value to the output variable “result”, which is then returned as the output of this function. Note that whenever consumption is zero or the liquidity constraint is violated, a large negative number is assigned to the output variable, “result”. In other words, a penalty is imposed for zero consumption choice and/or for violating the liquidity constraint when it is imposed, such that choices associated with these outcomes will never be optimal.

A scenario file contains input data for the program defined by the three functions described above. The scenario file (Appendix, Figures A4a & A4b) is organized into a series of statements each starting with a key word and ending with a semicolon. The file contains a MAX statement which indicates to ASDP that the objective is to maximize the optimal value function. The NO_CHANGE value specifies how many successive iterations must result in the same set of decisions before a stationary strategy is assumed to have been found while ALPHA is the discount factor (*i.e.* \( \beta = 0.8989 \)).

There are three STATE statements which define the state variables for wealth, production costs, and county-level revenue, respectively. The DECISION statements serve a similar function. The DISTRIB statements identify particular discrete probability density functions and names them (“revenue”, “cost”, and “farm”). These names are later used in the COMBINE statement to reference the density functions (i.e. “revenue” for county-level revenue, “cost” for production costs, and “farm” for farm-level revenue). Immediately following each DISTRIB statement is a RV statement that lists the random
variables defined by this distribution. Following the RV statements are EVENT statements. Each EVENT statement is followed by the probability of that event occurring and then the value that the random variable takes on if the event occurs. In this case, the events represent the discrete random shocks for county revenue, production costs, and farm revenue, respectively.

The STAGE statement comes next, followed by state variable increment sizes. This is followed by a COMBINE statement which specifies the ranges of the state and decision variables that will be evaluated and the probability distributions that will apply to the specified combination of the state and decision variables. In the COMBINE statement in this file, initial wealth states range from -$1000 to 2000, while the initial production costs and county-level revenue can only take the values, $160 and $362, respectively. Note that these are the starting values for production costs and county-level revenue, respectively, which will be passed on to the transition equations in the state dynamics function file. Next, the ranges of the decision variables are specified. Insurance coverage ranges from 0 to 0.85 in 0.05 increments and consumption ranges from $0 to $2000 in $20 increments. Finally, the distributions of the random variables are specified by referencing the previously defined distributions.

Given the state dynamics, stage return, and the terminal value function files, and the scenario file, the ASDP algorithm finds a solution to the optimization problem in the following steps.

Step 1: Optimal value function is initialized.

This step utilizes the terminal value function file (Appendix, Figure A1).
Step 2a: For every combination of state variables, decision variables, and random variables, a vector of next period’s \((t+1)\) state variables is computed.

This step utilizes the state dynamics function file (Appendix, Figure A2) and the scenario file (Appendix, Figures A4a & A4b).

Step 2b: For each of the period \(t+1\) variables computed in step 2a, an associated (joint) probability is assigned.

This step also utilizes the state dynamics function file (Appendix, Figure A2) and the scenario file (Appendix, Figures A4a & A4b).

Step 3: The optimal value function is computed.

This step utilizes the stage return function file (Appendix, Figure A3) and the results from steps 1 and 2. The set of values of the decision variables that maximize this function are the optimal decisions given the state variables at \(t\). These values are stored.

Step 4: The value function in step 1 is replaced with that computed in step 3 and the process is repeated until the convergence criterion (specified in the scenario file) is reached.
4.5 Model Validation

In this section the numerical model discussed above is validated by numerically solving the theoretical model in Chapter Three and checking results against those predicted for the theoretical model. The numerical model is taken to be credible if the results are consistent with the analytical results. For ease of exposition and clarity, the model presented in Chapter Three is referred to as the theoretical model while the one presented in this chapter is the numerical model.

As noted earlier, in order to calibrate the theoretical model, some changes to the calibration outlined above are needed. These changes can be summarized as follows:

1. Production costs and farm revenue are assumed to be iid processes. In the numerical model these were modeled as AR(1) processes. This implies that the transition equations for both variables are now given as a constant mean plus a random shock. The mean for farm revenue was taken to be $328 per acre while that for production costs was $160 per acre.

2. In the theoretical model the revenue trigger index, $k_r$, is the maximum possible revenue state that can be realized. This is based on the definition of the farmer’s realized crop revenue, $y_r$, which was assumed to be an iid random variable with a cumulative distribution function $F(\xi) = \text{Prob}[y_r \leq \xi]$, defined over the support $[a,b]$ so that $F(a) = 0$ and $F(b) = 1$. In the numerical model, however, the trigger index was computed as the expected revenue to be consistent with the way it is computed.
in practice. The maximum possible revenue was $585 per acre under the assumption of an iid process. Hence, $k_t$ was set at this figure.

3. The discrete state space for insurance coverage level was specified as a range of coverage choices ranging from 0 to 1 in 0.05 increments of the insured revenue. As discussed earlier, this specification differs from the specification in the numerical model where the maximum coverage level allowed is 0.85 of the insured revenue (under the individual revenue insurance design). Again, these differences arise because the theoretical model is based on a more general insurance problem while the numerical model attempts to mimic the way crop insurance is offered in practice.

With these modifications to the calibration, the model was solved assuming that the farmer is faced with: (1) actuarially fair insurance with complete credit markets; (2) actuarially unfair insurance with complete credit markets; and (3) actuarially fair insurance but the farmer faces a liquidity constraint.

The results presented in Figure 4 below show that when faced with actuarially fair insurance and no liquidity constraints, the farmer chooses to buy full insurance coverage at all levels of initial wealth. These results also show that when there is loading on the insurance premium, the farmer will choose to buy less than full coverage of actuarially fair insurance at all levels of initial wealth. In particular, when there is a 30% loading on the premium, the farmer chooses a coverage level of 75% of the insurable revenue. These two results are consistent with the analytical results obtained in Chapter 3 (Propositions 1 and 2). Discussion of the implications of these results is deferred until Chapter Five which
contains detailed discussion of the results of this study. The goal of this section is only to validate the numerical model.

Figure 4. Optimal Insurance Coverage with Complete Credit Markets

The results presented in Figure 5 show the optimal insurance decision of a farmer who faces a liquidity constraint. In particular, the farmer’s net wealth is restricted to be nonnegative in every period. These results show that when faced with a liquidity constraint, the farmer chooses less than full coverage of actuarially fair insurance at initial wealth levels in which the constraint is binding. In some cases, no insurance coverage is taken. However, if the insurance is subsidized, coverage level increases to full coverage.
for some of the cases in which the liquidity constraint is binding. The results also show
that the farmer’s insurance decision when the premium is loaded follows a similar pattern.
That is, at initial wealth levels for which the constraint is binding the farmer will choose to
reduce coverage to levels below those that would be expected in the absence of such a
constraint. These results are consistent with the analytical result presented as Proposition
3 in Chapter Three. Hence, these results confirm that the numerical model performs as
expected.

Other validation procedures were also carried out. During execution of the
dynamic programming model, various messages are written to the model log file. If there
are any errors, a message to that effect is written. The log file was examined and no errors
were found. In addition, the log file also provides intermediate steps of the optimization
process. These were also examined to check if the algorithm was carrying out these steps
correctly. In particular, a two-period model with only two states of nature was calibrated
and estimated using this algorithm and a solution obtained. Next, a solution to this
simplified model was manually computed. Finally, the two solutions and their
 corresponding intermediate steps were compared. They were found to be identical.
The conclusion is that the numerical model is being solved correctly. The next section
discusses the experimental designs under which the model was solved to generate the
results reported in Chapter Five.
Figure 5. Optimal Insurance Coverage with a Liquidity Constraint

4.6 Experimental Design

The numerical model was solved under two alternative insurance designs. First, optimal insurance choices are examined using an individual farm revenue insurance design that has no basis risk, and second these optimal choices are investigated to determine how they change under an area based insurance design where individual farms experience “basis risk”. Basis risk is said to exist whenever there is a difference between the farmer’s actual insured loss and the loss that is indemnified. For example, under the
RMA/USDA facilitated Group Risk Income Protection (GRIP) scheme, indemnity payments are made only when the average county revenue for the insured crop falls below the revenue chosen (insured) by the farmer. Payments are not based on the individual farmer’s loss. Therefore, individual revenue losses may fail to be indemnified if the average county revenue does not suffer a similar level of loss as the farmer. Conversely, a farmer may receive indemnity payments even if he/she did not suffer any loss provided that the average county revenue reflects a loss.

In this study, individual farm revenue and county-level revenues are related by equation (4.10) which, for convenience, is restated here as:

\[ y_{it}^f = \alpha_0 + \alpha_1 y_{it}^c + u_{it} \]

As defined earlier, \( t \) is a time subscript, \( i \) is an individual farm subscript, \( \alpha_0 \) and \( \alpha_1 \) are estimated parameters, \( u_{it} \) is the error term and superscripts \( f \) and \( c \) represent farm and county, respectively. Basis is defined as the difference between realized farm revenue and county-level revenue, i.e. \( \text{basis} = y_{it}^f - y_{it}^c \). On the other hand, basis risk is defined as the variance of the basis, i.e. \( \text{basis risk} = \text{var}(y_{it}^f - y_{it}^c) \). Substituting equation (4.13) in this expression and using the fact that the correlation between county revenue and the error term \( u_{it} \) is by design equal to zero yields the following definition for basis risk

\[ \text{Var}(y_{it}^f - y_{it}^c) = (\alpha_1 - 1)^2 \text{var}(y_{it}^c) + \text{var}(u_{it}) \]

Equation (4.14) implies that basis risk depends on \( \alpha_1 \), the variance of county-level revenue, and the variance of the contemporaneous shock to individual farm revenue. The
impact of alternative levels of basis risk, holding the variance of county-level revenue constant, can be examined by changing $\text{var}(u_t)$ and $\alpha_t$.

For convenience, the individual farm revenue design is referred to as the IR design while the area revenue insurance design is referred to as the AR design. In the IR design, the guaranteed revenue level is computed as a product of the expected farm yield and the expected harvest time cash price while, in the AR design it is computed as a product of the expected county-level yield and the expected harvest time cash price. The indemnification index (insurance payout trigger) for the IR design is realized farm revenue. Under the AR design, the insurance payout is triggered by county-level revenue. Note that there is a large number of revenue insurance contract designs and it would be impossible to investigate all of them. The experimental design here is a generic specification which, although it may not fit all the characteristics of these designs exactly, maintains the key features of these insurance schemes and allows one to investigate interaction between insurance, consumption smoothing, and liquidity constraints.

Under these two alternative insurance designs, transition equation (4.2) for next period’s liquid wealth becomes:

(4.15) $$w_{t+1} = (1+r)[w_t - x_t - c_t - P^\text{IR}(k^\text{IR}_t)] + \max(k^\text{IR}_t - y_t, f)$$

under the IR insurance design and,

(4.16) $$w_{t+1} = (1+r)[w_t - x_t - c_t - \theta P^\text{AR}(k^\text{AR}_t)] + y_t^f + \theta \max(0, k^\text{AR}_t - y_t^C)$$
under the AR insurance design, where $\theta$ is a scale factor while $f$ and $c$ are superscripts used to identify farm and county-level revenues, respectively. The transition equation in (4.15) represents a case where there is a one-to-one correspondence between the farmer’s insurance payout and the actual insured revenue loss. On the other hand, the transition equation in (4.16) represents the case where a potential mismatch between the insurance payout and the farmer’s insured loss exists. However, under the AR design the number of insured acres can be higher than the number of planted acres. That is the number of insured acres are computed as a product of the number of planted acres and a scaling factor, $\theta$. In practice is $\theta$ lies between 0.9 and 1.5 of planted acres. Further, while in the IR design coverage is restricted to 85% of insurable revenue, up to 90% coverage is permitted under the AR design.

Using these two designs, optimal insurance choices are examined under the following situations.

1. Insurance choice with no liquidity constraint and no loading on the insurance premium ($\lambda_t=0$ and $\alpha=0$);\textsuperscript{14}

2. Insurance choice with no liquidity constraint but with positive loading on the insurance premium ($\lambda_t=0$ and $\alpha>0$) to account for administrative costs and insurer profits;

\textsuperscript{14} As defined earlier, $\lambda_t$ is the Lagrange multiplier on the liquidity constraint and $\alpha$ is the loading factor on the insurance premium.
3. Insurance choice with no liquidity constraint but with negative loading on the insurance premium ($\lambda_t=0$ and $\alpha<0$) to account for a subsidy on insurance;

4. Insurance choice with a liquidity constraint but no loading on the insurance premium ($\lambda_t>0$ and $\alpha=0$);

5. Insurance choice with a liquidity constraint and a positive loading on the insurance premium ($\lambda_t>0$ and $\alpha>0$); and

6. Insurance choice with a liquidity constraint and a negative loading on the insurance premium ($\lambda_t>0$ and $\alpha<0$).

In addition, sensitivity analysis is used to investigate the effect of: (i) different levels of basis risk; (ii) different levels of loading on the insurance premium; and (iii) different levels of risk aversion on optimal insurance choice.

Situation 1 represents the base case in which insurance coverage is actuarially fair and capital markets are complete. Situation 2 represents a case in which the insurance coverage is actuarially unfair but capital markets are complete. The results from this situation are compared with the base case results to show the impact of a positive loading on the optimal insurance choice. In contrast to situation 2, the case represented by situation 3 shows the impact of a subsidy on the optimal insurance choice. Situation 4 represents a case where the insurance coverage is actuarially fair but capital markets are imperfect in the sense that a farmer is limited on how much he can borrow at the going interest rate. A comparison of the results under this situation with the base case shows the impact of a liquidity constraint on the optimal insurance choice. In situation 5, insurance
coverage is actuarially unfair and capital markets are imperfect. The results from this situation are compared with situations 2 and 4 to show the impact of the interaction between actuarially unfair insurance and imperfect capital markets on optimal insurance choice. Situation 6 represents a case in which insurance is subsidized in a world of imperfect capital markets. The results for this case are compared to situation 3 to show the impact of a subsidy on optimal insurance choice when capital markets are imperfect.

4.7 Summary

This chapter discussed the stochastic dynamic programming technique used in this study. Detailed explanations are provided on how all the discrete state and control variable spaces were specified as well as the assumptions made concerning these variables. The empirical data generating processes (DGPs) for production costs, county-level revenue, and farm-level revenue for a representative farm in Adair County, South West Iowa, are also discussed. In particular, econometrics procedures are used to find the models which are then used to define the variable state space.

The solution algorithm (ASDP) was also described. Program and data files are presented and discussed, providing a flavor for how the algorithm works. In addition, the credibility of the numerical model is investigated. The numerical model validation results were compared to, and found to be consistent with, the analytical results of Chapter Three. Other validation procedures are also discussed. The conclusion from the validation results was that the numerical model is solving the farmer’s problem correctly.
Finally, several experimental designs for model solutions are discussed. Revenue insurance contracts are generally offered under a variety of designs depending upon how the insurance trigger is calculated. Two alternative designs that were used in this study are discussed. These are the individual revenue (IR) and the area revenue (AR) insurance designs. The former is an example of an insurance design without basis risk while the latter is a design with basis risk. Hence, the use of these two designs facilitates the investigation of the impact of basis risk on optimal insurance choice. The way basis is defined in this study is explained and an explicit expression for basis risk is provided.

Finally, six experiments are described that were used to investigate the effect of liquidity constraints and loading of the insurance premium on the farmer’s optimal insurance choice. Starting with a base case in which insurance coverage is actuarially fair and capital markets are perfect, various assumptions are progressively relaxed to provide alternative experimental designs. The assumption of actuarially fair insurance is relaxed in order to examine the impact of premium loading and a subsidy on the optimal choice. Finally, the assumption of perfect capital markets is relaxed to investigate the impact of a liquidity constraint.
CHAPTER FIVE

NUMERICAL RESULTS

5.1. Introduction

This chapter reports the optimal insurance coverage rules for a representative corn farmer from Adair County, South West Iowa. The stochastic dynamic programming model is solved for the IR and AR insurance designs discussed in Chapter four. As discussed earlier, these designs differ on how the insurance payout is triggered, the maximum coverage levels permitted in practice, and the number of permitted insured acres relative to planted acres. In the IR insurance design, the insurance payout trigger is realized farm revenue while, in the AR insurance design it is county-level revenue. Therefore, the AR insurance design has basis risk while the IR insurance design does not. However, under the AR design insured acres can be higher than planted acres by as much as 1.5. In addition, coverage levels of up to 90% of insurable revenue are permitted while under the IR design, coverage is restricted to a maximum 85% of insurable revenue.

The model is solved under the IR design assuming complete credit markets exist in order to investigate the optimal insurance choice when: (1) the insurance premium is actuarially fair; (2) the insurance premium is actuarially unfair with positive loading; and (3) the insurance is subsidized (negative loading on the premium). Next, the assumption of complete credit markets is relaxed in order to investigate the impact of a liquidity constraint on the optimal decision again, under the IR design. Finally, the model is solved
for the AR design in order to investigate the impact of basis risk on optimal insurance choice.

The rest of the chapter is organized as follows. Section 5.2 provides the results on optimal insurance coverage under the IR design assuming complete credit markets. The optimal insurance decision when insurance is actuarially fair is presented and then compared with the decision when there is a premium loading. In Section 5.3, the assumption of complete credit markets is relaxed by introducing a liquidity constraint. Again, the impact of the liquidity constraint is examined under a variety of assumptions, concerning the actuarial fairness of the insurance premium. Section 5.4 reports the results on optimal choice under the AR design under a variety of assumptions while Section 5.5 contains a sensitivity analysis on the parameters of the model. Finally, Section 5.6 provides a summary of the chapter.

5.2. Optimal Insurance Choice under the IR design with no Liquidity Constraint

This section presents results on optimal insurance choice under the individual farm revenue insurance design (IR design) and no liquidity constraint. It is assumed that the farmer uses insurance to manage revenue risk and can also borrow and save to further smooth consumption. A lifetime borrowing constraint requires that the farmer must eventually pay back all that is borrowed, so that current consumption cannot be financed indefinitely by borrowing more and more money. In finite horizon models, in the last period the farmer cannot borrow and must pay back all previous loans. With the same reasoning, the farmer does not save in the last period because he cannot derive any value
from such savings. For infinite horizon models, this constraint is imposed as a transversality condition which requires that the limit of the expected discounted wealth be zero (see Chapter four, Equation 4.5).

Figure 6 shows the optimal insurance coverage choice as a function of the farmer’s liquid wealth available at the beginning of the period (planting time). These results show that, with no liquidity constraint, the farmer takes the maximum allowable coverage of actuarially fair insurance at all current net wealth levels. In particular, the farmer chooses coverage of 0.85 of insurable revenue. This result is consistent with the standard static solution in which the farmer always takes full coverage of actuarially fair insurance. It is also consistent with the analytical result derived in Chapter three for a farmer faced with actuarially fair insurance and facing no liquidity constraint (Proposition 1). This result implies that with complete credit markets, one would expect farmers to always take the maximum allowable coverage of actuarially fair insurance as protection for unanticipated revenue losses.

Figure 6 also shows the optimal insurance coverage choice for a farmer who is faced with actuarially unfair insurance. Specifically, these results pertain to a 30% and a 60% premium loading above the actuarially fair level. The results show that at moderate levels of insurance premium loading, such as 30%, the farmer continues to take the maximum allowable coverage at all initial wealth levels. As one might expect, however, the optimal coverage level falls as the loading factor rises. In particular, with a 60% loading the farmer’s optimal level of insurance coverage is reduced from the maximum
allowable coverage level of 85% to 75% for initial wealth levels that lie between -$260 and $ 40 per acre and, to 70% for initial wealth levels above $40 per acre.

Figure 6. Optimal Coverage of individual farm revenue Insurance for a Farmer Facing no Liquidity Constraints.

An explanation for this result is that in the numerical model, maximum allowable coverage is 85% of the insurable revenue because this is the maximum coverage in practice. The model was re-estimated under a scenario in which 100% of the insurable revenue could be covered. Under this scenario any positive premium loading reduces coverage below the full coverage level. This includes the case in which the loading factor
is 30% above the actuarially fair level. This finding is consistent with the well known result from insurance theory that, a positive premium loading leads to lower than full insurance coverage. It is also consistent with the analytical result in Chapter three \((Proposition ~2)\) in which it was shown that the optimal coverage level falls below the full coverage level as the premium loading rises above the actuarially fair level. However, the results in Figure 6 provide additional insights concerning the farmer’s optimal insurance choice that could not have been obtained from the analytical results alone or with static models.

First, the results suggest that with premium loading factors at moderate levels (e.g. 30%), and with no liquidity constraints, one would expect farmers to take the maximum of 85% coverage that is allowed in practice. Therefore, as long as credit markets are complete a moderate premium loading would not appear to be a good explanation for weak participation in insurance programs.

Second, only when the premium loading is high (e.g. 60%), would one expect wealthy farmers to take less than the maximum coverage allowed of 85%. This suggests that when insurance is (very) expensive, only farmers who are already in debt would be willing to take more loans to finance insurance. As they accumulate wealth, farmers substitute savings for expensive insurance in managing revenue risk. The explanation for this result is based on viewing insurance and savings as substitutes for mitigating the impact of revenue risk on consumption. In this view, the explanation is that the opportunity cost (measured in terms of consumption forgone) of not insuring potential revenue loss is much higher for farmers already in debt than for those with positive initial
wealth. Hence, they are more likely to choose the maximum allowable coverage even if the policy is expensive.

These results show that the standard results from static insurance models also hold within a dynamic framework that explicitly takes into account the farmers’ consumption smoothing strategies. In particular, these results have shown that as long as complete credit markets exist and the farmer can freely lend and borrow at the risk free rate, then consumption smoothing plays no role in his/her insurance decision, except when insurance is very expensive.

5.3 Optimal Insurance Choice under the IR design with a liquidity constraint

In this section results are presented assuming the farmer faces a liquidity constraint. In particular, the model is solved assuming that the farmer’s wealth is restricted to be non-negative. Implying that being a net borrower is prohibited. The results under this assumption are presented in Figure 7 below.

These results show that a farmer facing a liquidity constraint will take the maximum allowable coverage of actuarially fair insurance only at wealth levels in which the constraint is non-binding. When the liquidity constraint is binding the farmer cannot borrow to finance production costs, consumption and the purchase of insurance.

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\(^{15}\) It is a straightforward matter to allow for net borrowing. However, the results are essentially similar to those obtained here when the constraint is set to zero rather than some negative net wealth level.
Therefore, he chooses not to insure because current consumption is too valuable. In other words his marginal propensity to consume out of current wealth is equal to one. This is because the CRRA utility function makes consumption infinitely valuable as consumption approaches zero. Therefore, the farmer is willing to trade off anything (including insurance) in order to be able to consume more at low wealth levels because he cannot borrow to finance consumption. This result is consistent with the analytical result.
presented in Chapter three (Proposition 3) which showed that if farmers are faced with a liquidity constraint, then even if the insurance is actuarially fair, they may choose reduced or even zero coverage, depending on the severity of the constraint.

When insurance is actuarially unfair, the results follow a similar pattern as described above. That is, at levels of initial wealth in which the liquidity constraint is binding, no insurance coverage is taken. However, in contrast to the results presented in Figure 6 (with no liquidity constraint), these results show that the liquidity constraint causes farmers to take the maximum allowable coverage of actuarially unfair insurance at some positive wealth levels. In particular, with a 60% loading on the premium, maximum allowable coverage is taken at initial wealth levels between $200 and $780 per acre. As was seen in Figure 6, with no liquidity constraint less than the maximum allowable coverage of actuarially unfair insurance was taken at all positive initial wealth levels.

A binding liquidity constraint limits (or eliminates) the farmers ability to diversify downside risks over time. As shown by Gollier (2001) for the DARA utility function used in this study, a liquidity constraint induces more risk aversion, *ceteris paribus*. Specifically, at low initial wealth levels, Gollier (2001) provides an intuition for the result as follows: “the liquidity constraint is likely to be binding for poorer people, which makes them more risk-averse because of their inability to time diversify risk” (p.274). In contrast, wealthier people are in a better position to smooth shocks to their income over time, because they are less likely to be liquidity constrained in the near future (Gollier 2001, 2003). More generally, following Epstein’s (1983) study, Gollier (2001) shows that within a continuous-time, infinite horizon framework, all value functions exhibit decreasing
absolute risk aversion. This result forms the basis for his intuition that “a larger wealth better insulates the consumer from the liquidity constraint” (p.277) and, hence, reduces his aversion to risk. Further, he shows that in the continuous-time framework this effect dominates even when the consumer exhibits increasing absolute risk aversion.

Overall, these results have two implications. First, they suggest that a liquidity constraint may cause farmers to reduce coverage below the full insurance level even for actuarially fair insurance and, depending on the severity of constraint no insurance may be taken. Second, they suggest that a liquidity constraint induces farmers to behave as if their degree of absolute risk aversion is much more decreasing with respect to wealth than would be expected without a liquidity constraint. This inference is especially evident when insurance is actuarially unfair. In this case, the optimal insurance choice was to take the maximum allowable coverage at lower (positive) initial wealth levels because the farmer is much more risk averse in these ranges of wealth.16 At higher levels of wealth, the solution reverts to that obtained in the absence of a liquidity constraint because the farmer is less risk averse in those ranges of wealth, ceteris paribus.

The optimal choice was also investigated when insurance is subsidized. The results obtained are trivial and are therefore not presented here. In particular, with no liquidity constraint, the maximum allowable coverage of subsidized insurance is taken. This is similar to the optimal coverage choice when insurance was actuarially fair. When there is a liquidity constraint however, the subsidy enables farmers to take insurance at some

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16 With no liquidity constraint, the maximum allowable coverage was not optimal at all positive wealth levels for insurance with a 60% premium loading (see Figure 6).
lower wealth levels than was done with actuarially fair insurance. This is equivalent to a slight shift to the left of the graph in Figure 7 for actuarially fair insurance. In other words, the subsidy somewhat relaxes the severity of the liquidity constraint although, still at the constraint no insurance will be taken. Next, the impact of basis risk on the optimal insurance choice is examined.

5.4 Insurance Choice under the AR design with no Liquidity Constraint

This section presents results on optimal insurance choice under the area-based revenue insurance design (AR design). As discussed earlier, the AR insurance design used in this study is a design with basis risk which exists because indemnification is based on county-level revenue and not the farmer’s realized revenue. Therefore, to the extent that an individual farmer’s revenue is not perfectly (positively) correlated with the county-level revenue, this insurance may fail to protect that farmer’s revenue losses. In other words, it is possible to have a situation in which the farmer incurs a loss but this loss is not indemnified because the county-level revenue does not trigger indemnity payments.

As can be seen in Figure 8, when there is basis risk the farmer fails to take the maximum allowable insurance coverage at some wealth levels, even if insurance is actuarially fair and there is no liquidity constraint. Specifically, less than the maximum allowed coverage of actuarially fair insurance is taken except for farmers heavily in debt.

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17 The graph is similar to that in Figure 5 (chapter Four) with a maximum allowable coverage of 0.85 instead of 1.
Figure 8. Optimal coverage of actuarially fair insurance under the AR design with no liquidity constraint when the correlation between farm-level and county-level revenue is 0.63

It appears that the presence of basis risk discourages farmers with high initial wealth from purchasing insurance even when it is actuarially fair. There are several possible factors that may be driving this rather surprising result.

As discussed in Chapter 4 (Section 4.6), basis risk depends on the variance of county-level revenue, the variance of the contemporaneous shock to individual farm revenue, and the parameter relating county-level revenue to individual farm revenue, \( \alpha_j \).
That is, under the area based insurance scheme, the producer’s risk as measured by the variance of net revenue depends on the variances of county-level and farm-level revenues. In an important paper on area-yield crop insurance, Miranda (1991) shows that the amount of risk reduction under the area-based scheme depends highly on the correlation between a producer’s yield and the area yield. In particular, he shows that the higher the correlation between area yield and the producer’s yield, the greater the risk reduction under the area based scheme. He also shows that if the number of acres covered are not restricted to farmed acres, then the producer can achieve maximum risk reduction by selecting an optimal scaling factor. Further, he shows that the optimal scaling factor also depends on the correlation between farm and area yields.

To summarize, the results in Figure 8 are driven by at least two key factors: the correlation between county-level and farm revenues and the scale factor of the number of insured acres to farmed acres. The result shown in Figure 8 pertains to a correlation of 0.63. In addition, scaling factors of 0.9 - 1.5 with increments of 0.1 were allowed for in the model. Although not presented in the figure, the optimal scale ranged from 0.9 to 1.4 at various levels of initial wealth and optimal coverage. Similar to optimal coverage levels, the optimal scale was also found to decrease with initial wealth.

The preceding discussion implies that under the AR design, basis risk exposes insurers to additional risk which may preclude them from taking the maximum allowable coverage even if insurance is actuarially fair. An intuition for this result is that the uncertainty of not receiving indemnity payments when insured losses have been incurred becomes a disincentive for purchasing this type of insurance for wealthy farmers.
Within an insurance-consumption smoothing framework the presence of basis risk and the CRRA utility function can lead to the result above. Compared to the IR design, basis risk in the AR design is akin to an increase in risk, *ceteris paribus*. In other words, the variability of consumption under the AR design is higher than in the IR design. As shown by Leland (1968) and Sandmo (1970) for an additively separable utility concave function with a positive third derivative, an increase in uncertainty increases the precautionary demand for saving. This is because a positive third derivative of a concave utility function implies that the marginal utility function is convex and hence, the expected marginal utility increases when the variability of its argument (consumption) increases. In subsequent studies Rothschild and Stiglitz (1970) provide a more general definition of increasing risk and show that the result by Leland (1968) and Sandmo (1970) holds (Rothschild and Stiglitz; 1971). Further, Kimball (1990) confirms this result by applying the analytical framework of Rothschild and Stiglitz (1971) to the two-period savings problem under income uncertainty considered by Leland and Sandmo. He shows that if the marginal utility function is convex, an increase in risk will decrease optimal consumption and increase optimal saving in the first period.

The CRRA utility function used in this study is also found in Rothschild and Stiglitz (1971). Its specification as \( U(C) = C^{(1-\gamma)/(1-\gamma)} \), where \( \gamma \) is the constant coefficient of relative risk aversion, implies that it has a positive third derivative if \( \gamma > 0 \). For the results above, \( \gamma = 2 \). Therefore because \( u'''' > 0 \), the CRRA function has precautionary demand for saving and since \( \lim_{c \to 0} U' = \infty \), optimal consumption is bounded away from negative or zero consumption. Also, for this utility function specification, risk aversion
and precautionary demand for saving are controlled by the same parameter, $\gamma$ (Rothschild and Stiglitz, 1971).

To summarize, under the area based scheme basis risk exposes insurers to a residual uninsurable risk which may preclude them from purchasing insurance even if it is actuarially fair and there is no liquidity constraint. Basis risk is similar to an increase in risk when compared to the IR design. This in turn increases the farmer’s precautionary demand for accumulating wealth because of the CRRA utility function used in this study. However, the increase in precautionary demand for saving comes at the expense of insurance. Furthermore, for the type of utility function used here risk aversion and demand for precautionary saving are controlled by the same parameter, $\gamma$. For the model in this study it seems that the effect of the precautionary motive for accumulating wealth dominates demand for insurance. An intuition for this finding is that when a farmer is faced with an insurance design with basis risk (residual uninsurable risk), he would like to accumulate enough wealth in order to self-insure. That may be the reason why he takes out the maximum allowable coverage at low levels of initial wealth. Once he has accumulated enough wealth he resorts more to using a self-insurance strategy rather than formal insurance.

However, because the amount of basis risk depends on the correlation between farm revenue and county-level revenue, one would expect the optimal insurance decision to be different for different correlations. Figure 9 shows the optimal insurance choice under the AR design with a correlation of 0.9 (between farm revenue and county-level revenue) and a scale factor of 1.5. These results were obtained by re-estimating the model
using shocks to farm revenue that give rise to a correlation of 0.9, holding the variance of county revenue and $\alpha_1$ constant. As expected, these results show that at a higher correlation between farm-level and county-level revenues, higher levels of insurance coverage are taken. In particular, when the insurance coverage is actuarially fair, the maximum allowable coverage is taken at all initial wealth levels. The reason for this is that at high correlations basis risk is lower than at low correlations, holding other factors constant. In other words, there is lower residual uninsurable risk and risk aversion dominates the precautionary demand for saving causing the maximum allowable coverage to be taken at all wealth levels.
When the insurance is actuarially unfair, these results show that less than the maximum allowable coverage is taken at higher wealth levels. As can be seen from Figure 9 the combined effect of basis risk and premium loading further reduces the optimal insurance coverage level when compared to the results presented in Figure 6 (with no basis risk). For example, when there is a 60% loading on the premium, the optimal coverage level falls below the maximum allowable coverage level at all wealth levels above -$420 per acre and, to zero above -$160 per acre. This suggests that only when a farmer is highly in debt, is he willing to take loans to finance expensive insurance when
there is basis risk. At higher wealth levels, he substitutes savings for expensive insurance in managing revenue risk. It seems that at lower loading levels (e.g. 30%) the cost of insurance is not large enough to justify the building of buffer stock savings to self-insure. In other words, the price elasticity of insurance demand is low but increases with each additional increase in the loading factor and, at 60% loading it is high enough to induce a switch from formal insurance to self-insurance. Of course, this argument holds even in the absence of basis risk. However, it appears that there is more sensitivity to the cost of insurance when there is basis risk.

Next, the optimal insurance choice when the insurance is subsidized under the AR design is examined. With a correlation of 0.9 between farm-level and county-level revenue, the result is trivial because the maximum allowable coverage is taken at all wealth levels as long as the insurance is actuarially fair. Therefore, similar results are obtained with a subsidy. However, as shown earlier, less than the maximum allowable coverage is taken when the correlation is 0.63. Therefore, the impact of a subsidy is examined for the base case presented in Figure 8 above. These results are presented in Figure 10 below.
As can be seen from these results, when there is a 10% subsidy on the insurance premium the farmer takes the maximum allowable coverage at initial wealth levels of $900 per acre and below. Above that level of initial wealth, coverage is reduced below full coverage. As expected the subsidy increases the amount of insurance purchased. In the absence of a subsidy (Figure 8), less than the maximum allowable coverage of actuarially fair insurance was taken for initial wealth levels above -$360 per acre. With a 10% subsidy, the wealth level at which reduced coverage is taken is much higher ($900 per acre). However, the results show that, depending on the level of the subsidy, in some cases
it may still be optimal not to take the maximum allowable coverage insurance when there is basis risk. Nonetheless, these results suggest that subsidies can be effective in mitigating the reduced participation caused by the presence of basis risk, especially if the correlations between farm-level and county-level revenues are low. As shown in Figure 10 when the subsidy is high enough, say at 20%, then the optimal insurance choice is to take the maximum allowable coverage at all initial wealth levels. Thus, the effect of basis risk is completely eliminated at this level of subsidy.

The impact of a liquidity constraint on the optimal choice under the area based scheme (AR design) was also examined assuming a correlation of 0.9 between farm-level and county-level revenues. Similar to the individual farm revenue insurance design (IR design) the results showed that when the liquidity constraint is binding, the farmer may choose not to insure because current consumption is too valuable. Once the liquidity constraint becomes non-binding the farmer takes the maximum allowable coverage of actuarially fair insurance. The optimal insurance choice when insurance is actuarially unfair was found to follow a similar pattern as that seen under the IR design. That is, the liquidity constraint causes farmers to take the maximum allowable coverage of actuarially unfair insurance at some positive wealth levels in which, less than the maximum allowable coverage would be expected to be taken in the absence of the liquidity constraint. The intuition for these results has been discussed above.
5.5 Sensitivity Analysis

In order to learn how these results change with changes in the parameters of the model, sensitivity analysis were performed. The effect of changes in the premium loading have already been examined and discussed as they were part of the experimental design. The effect of basis risk at alternative correlations between farm-level and county-level revenues was also examined. In this section sensitivity analysis is conducted to determine the effect of changes in risk aversion and the farmer’s rate of time preference.

For the results reported above, the coefficient of risk aversion was set at $\gamma = 2$. The effect of an increase in risk aversion was investigated by re-estimating the model when $\gamma = 1.2, 1.5, 1.8, 2.2, 2.5, 2.8$ and $3$, and then the results were compared with the benchmark results when $\gamma = 2$. For $\gamma > 3$ the results were similar to those for $\gamma = 3$. For investigation, the model with basis risk was re-estimated assuming a correlation of $0.63$ between farm and county-level revenues and no liquidity constraint. The insurance design with basis risk was used because the effect was more discernible under this design. Virtually, identical results were obtained when $\gamma \neq 2.2$ and when $\gamma = 2.5$ or $2.8$, respectively. The results showed that insurance coverage was reduced in the latter case relative to the benchmark. When $\gamma = 3$ no insurance was taken.

Within an insurance-consumption smoothing framework one would expect an increase in risk aversion (the utility function becomes more concave) to have two effects. First, by Jensen’s inequality and, because of the increase in risk aversion one would expect the demand for insurance to increase. Second, one would expect an increase in precautionary demand for savings because of the CRRA utility function. However, the
increase in precautionary demand for savings comes at the expense of insurance. Therefore, a priori, the net effect of the increase in risk aversion is ambiguous. Furthermore, for the type of utility function used here risk aversion and demand for precautionary savings are controlled by the same parameter, $\gamma$. In this case it seems that the effect of the precautionary motive for accumulating wealth dominates demand for insurance.

Finally, the effect of changing the farmer’s measure of time preference parameter from the benchmark value $\beta=0.8989$ to 0.94 and 0.99, respectively was investigated. This is equivalent to changing the discount rate from 11.25% to 6.4% and 1%, respectively. In other words, the farmer is relatively more patient in the sensitivity analysis scenarios than in the base case. One would expect this decrease in the farmer’s rate of impatience to generate an increase in desire to accumulate wealth (precautionary savings) and as a consequence, (indirectly) reduce demand for insurance. The results showed this to be marginally true under the area-based scheme. Otherwise the results were essentially identical. Under the IR design, the results did not change.

These sensitivity analysis results show that the main results are robust at different levels of the parameters of the model. However, the results are somewhat sensitive to the level of risk aversion, especially under the area-based scheme.
5.5. Summary

The dynamic stochastic model developed in Chapter four was solved numerically to obtain the optimal insurance coverage choices for a representative corn farmer from Adair county, South West Iowa. The model was solved both for the case in which perfect credit markets exist and for the case in which the farmer faces a liquidity constraint. In addition, the model was solved using both an insurance design without basis risk and an alternative design with basis risk. Further, the effects of consumption smoothing and liquidity constraints were investigated under a variety of assumptions concerning the actuarial fairness of the insurance.

The results obtained show that with complete credit markets and no basis risk the farmer will take the maximum allowable coverage of actuarially fair insurance as protection against unanticipated future revenue shortfalls. However, if insurance is expensive because of a premium loading the farmer will reduce coverage below the actuarially fair level for some initial wealth levels. In particular when the insurance cost is as high as 60% above the actuarially fair level, the farmer will only take the maximum allowable coverage if his initial wealth is below -$360 per acre. In this case the farmer is highly in debt and because this debt must eventually be repaid, the farmer seeks to accumulate wealth to repay his loans and then substitute savings for expensive insurance to manage revenue risk.

The impact of a liquidity constraint on the farmer’s optimal choice of insurance coverage was also examined. The findings were that if farmers are faced with a liquidity
constraint, then even if the insurance is actuarially fair, they may choose reduced or even zero coverage, depending on the severity of the constraint.

The results also show that even with complete credit markets, basis risk in the insurance design may cause the farmer to choose less than the maximum allowable coverage of actuarially fair insurance at some initial wealth levels. Under this design there is residual risk that is uninsurable. Therefore, the uncertainty of not receiving indemnity payments when insured losses have been incurred becomes a disincentive for purchasing this type of insurance for wealthy farmers. Instead, such farmers prefer self-insurance. Hence, depending on the correlation between the farmer’s revenue and county-level revenue the resultant basis risk under this design may discourage farmers from purchasing insurance.

The results also show that the impact of basis risk and/or high insurance costs may be mitigated by subsidizing insurance. The findings of this study suggest that subsidies in the range of 20% or so are sufficient to cause farmers to take the maximum allowable coverage even when basis risk is high (equivalent to a correlation of 0.63).

Finally, sensitivity analysis found that as risk aversion increases it generates an increase in the farmer’s motive for holding precautionary savings at the expense of insurance demand, especially under the area-based scheme. This finding is, in part, dependent on the nature of utility function used. In this study a CRRA utility function is used in which risk aversion (hence, demand for insurance) and the demand for precautionary savings are determined by the same parameter. For the specifications of the model in this study, the demand for precautionary savings seems to dominate demand for
insurance. Finally, the sensitivity analysis showed that the results of this study were only marginally sensitive to the farmer’s rate of time preference.

The conclusion from the sensitivity analysis is that results are robust at different levels of the model parameters although, they are somewhat sensitive to the farmer’s degree of risk aversion, especially if there is basis risk in the insurance design.
CHAPTER SIX

SUMMARY AND CONCLUSION

The goal of this study was to determine the effect of credit constraints on optimal revenue insurance choices for a risk averse farmer who uses insurance to manage risk and also borrows and saves to further smooth consumption. The main contribution of this study to existing work on crop insurance, was to examine the farmer’s crop insurance problem in a dynamic framework with liquidity constraints. To date, only one study (Atwood et al. 1996) has examined the use of agricultural insurance instruments in a dynamic framework. However, neither revenue insurance decisions nor the effect of liquidity constraints were considered in that study.

In this study, the effects of consumption smoothing and liquidity constraints were investigated by examining optimal insurance choice under a variety of assumptions concerning the insurance premium schedule. This was accomplished using a dynamic consumption model in a time separable expected utility framework. Dynamic programming was used to study optimal decision rules for insurance coverage choices of a representative corn farmer from Adair County in Southwest Iowa.

Owing to the paucity of studies directly addressing the issues considered in this study, it was necessary to first provide a basic relationship between consumption smoothing and insurance. This was accomplished through a review of selected studies from developing as well as developed countries. This review was presented in chapter
Two. Three conclusions emerged from the development literature. First, there is empirical evidence that households in developing countries use various coping strategies to smooth consumption including off-farm employment, use of risk-decreasing inputs, informal credit and insurance arrangements, and livestock holdings. Second, because credit and insurance markets are missing, imperfect, or simply inaccessible, farmers in these countries use a variety of non-market risk sharing and credit provision arrangements such as, informal credit and insurance mechanisms, rotating savings and credit associations, and extended family networks. Third, the interaction between consumption smoothing and (formal) crop insurance is not covered in that literature, perhaps because of lack of crop insurance markets in developing countries.

Similarly, from the literature in a developed country setting where complete credit and insurance markets are often assumed to exist, there were two main findings. First, there is empirical evidence that in general U.S. households smooth consumption through borrowing and saving. Second, and more important, some households face borrowing constraints which limit their ability to smooth consumption at desired levels. Given this, it is inferred that a possible relationship exists between borrowing and crop insurance decisions because borrowing constraints affect consumption decisions and, therefore, they should also affect crop insurance decisions.

Further, the contribution of this study to the broader crop insurance literature was clarified. Upon reviewing studies on yield and revenue insurance the (relevant) finding was that both yield and revenue insurance have primarily been studied using static models. This study conjectured that analyzing the revenue insurance problem in a dynamic setting
could potentially shed new insights on farmers’ inter-temporal risk management behavior. This conjecture was based on inferences drawn from the literature on hedging, where both static and dynamic models have been extensively studied and used to derive hedging rules.

Chapter Three developed a conceptual dynamic model for studying the revenue insurance behavior of a crop farmer whose objective is to maximize the expected utility of lifetime consumption. Further, the assumptions needed for the model to be solved were specified and the sufficient and necessary conditions which characterize a solution to the farmer’s dynamic optimization problem derived.

Next, the effects of consumption smoothing and liquidity constraints were investigated by examining optimal insurance choice under a variety of assumptions concerning the insurance premium schedule. Three analytical results were obtained which can be summarized as follows. First, with no liquidity constraints, a risk-averse farmer will choose full coverage of actuarially fair insurance. This is consistent with the familiar result from static insurance theory that risk averse agents facing actuarially fair premiums will take full insurance coverage.

Second, with no liquidity constraint a positive loading on the insurance premium reduces the optimal coverage level below full coverage. With this result, this study also showed that another standard result from static insurance theory, that optimal coverage decreases with increases in the loading factor, also holds in a dynamic model with consumption smoothing and no liquidity constraints.

With these two results, this research showed that in a dynamic model with no liquidity constraints, insurance choices are not influenced by the desire to smooth
consumption, as long as complete and well-functioning credit markets exist that permit efficient consumption smoothing to take place.

Third, even if insurance is actuarially fair, a binding liquidity constraint reduces the optimal coverage below the full coverage level. This result shows that if farmers are faced with a liquidity constraint, even if insurance is actuarially fair, they may choose reduced or even zero coverage, depending on the severity of the constraint. The implication of this result is that, a binding liquidity constraint may cause farmers to purchase insurance less often than would be expected in the absence of the constraint.

In chapter Four, the stochastic dynamic programming technique used to numerically solve the farmer’s optimization problem was described. The solution algorithm (ASDP), was also described as well as the data used. Noting that revenue insurance contracts are generally offered under a variety of designs depending upon how the insurance trigger is calculated, two alternative designs were considered. The individual revenue (IR) and the area revenue (AR) insurance designs. The former design has no basis risk while the latter includes basis risk. In addition, six experiments used to investigate the effect of liquidity constraints and premium loading on the farmer’s optimal insurance choice were discussed. Starting with a base case in which insurance coverage is actuarially fair and credit markets are perfect, assumptions were progressively relaxed: first, the assumption of actuarially fair insurance, and second, the assumption of perfect credit markets to generate alternative situations faced by farmers.

Further, details were provided on how all the discrete state and control variable spaces were specified as well as the assumptions made concerning these variables. In
addition, detailed discussions on the empirical data generating processes for production costs, county-level revenue, and farm-level revenue were also provided. Production costs and county-level revenue were modeled as AR(1) processes while farm-level revenue was generated by scaling county-level revenue. The assumption here was that farm-level revenues would generally be related to county-level revenue because farmers in a given county face similar technology, weather conditions, and prices. An overview of the solution algorithm was also provided. Program and data files were presented and discussed to provide a flavor for how the algorithm works. Finally, the credibility of the numerical model was investigated. This was accomplished by comparing the numerical model validation results to the analytical results obtained in chapter Three and these were found to be consistent. Consequently, it was concluded that the numerical model was solving correctly.

In chapter Five, the stochastic dynamic programming model was solved numerically to obtain optimal insurance coverage choices. Furthermore, the model was solved both for the case with complete credit markets and for the case in which the farmer faces a liquidity constraint. Furthermore, the model was solved using both insurance designs with and without basis risk. Several numerical results were obtained with insights concerning farmer’s risk management behavior that can be summarized as follows.

As long as there no basis risk, complete credit markets exist, and the farmer can freely lend and borrow at the risk free rate, then consumption smoothing plays no role in his/her insurance decision except when insurance is very expensive. Therefore, the standard results from static insurance models also hold within a dynamic framework that
explicitly takes into account the farmers’ consumption smoothing strategies. More specifically, this research shows that with complete and well functioning credit markets: (i) the maximum allowable coverage of actuarially fair insurance will always be optimal; (ii) at moderate premium loading factors (e.g. 30%), the maximum of 85% coverage that is allowed in practice will still be optimal; and (iii) at relatively high (e.g. 60%) premium loading factors the maximum allowed coverage will no longer be optimal at most initial wealth levels. In particular, only when the farmer is highly in debt is the maximum allowable coverage optimal. At higher levels of initial wealth one would expect farmers to substitute savings for (expensive) insurance in managing revenue risk. The economic rationale behind this finding is that, because the lifetime constraint requires that all loans must be eventually repaid, it causes the farmer to act as if he is more risk averse when highly in debt than when he has less debt or positive initial wealth and, therefore, he takes relatively higher insurance coverage. In other words, the opportunity cost (measured in terms of consumption to be forgone) of not insuring potential revenue loss is much higher for farmers who are heavily in debt than for those with positive initial wealth. Hence, they are more likely to choose the maximum allowable coverage even if the policy is expensive.

This research also shows that even if there is no basis risk, if farmers are faced with a liquidity constraint, then even if the insurance is actuarially fair they may choose reduced or even zero coverage, depending on the severity of the constraint. This is because the presence of a liquidity constraint affects the farmer’s consumption behavior thereby affecting his insurance choice as well, and depending on its severity the farmer
may choose not to buy it. That is, a liquidity constraint causes the farmer to reduce coverage below the full insurance level even for actuarially fair insurance and, depending on the severity of constraint no insurance may be taken. As it appears the liquidity constraint induces farmers to behave as if their degree of absolute risk aversion is much more decreasing with respect to wealth than would be expected without a liquidity constraint. This inference is especially more evident when insurance is actuarially unfair. In this case, the optimal insurance choice was to take the maximum allowable coverage at lower initial wealth levels because the farmer is much more risk averse in these ranges of wealth. At higher levels of wealth, the solution was similar to the one with no liquidity constraint because the farmer is less risk averse in those ranges of wealth, *ceteris paribus*.

This study also found that under the area based scheme, basis risk exposes insurers to a residual uninsurable risk which may preclude them from purchasing insurance even if it is actuarially fair and there is no liquidity constraint. Basis risk is similar to an increase in risk when compared to the IR design. This in turn increases the farmer’s precautionary demand for accumulating wealth because of the CRRA utility function used in this study. However, because the increase in precautionary demand for saving comes at the expense of insurance, coverage is reduced. Furthermore, for the type of utility function used in this study risk aversion and demand for precautionary saving are controlled by the same parameter, $\gamma$. For this case it seems that the effect of the precautionary motive for accumulating wealth dominates demand for insurance. An intuition for this finding is that when a farmer is faced with an insurance design with basis risk (residual uninsurable risk), he would like to accumulate enough wealth in order to self-insure. That may be the reason
why he takes out the maximum allowable coverage at low levels of initial wealth. Once he has accumulated enough wealth he resorts more to a self-insurance strategy.

However, because the amount of basis risk depends on the correlation between farm revenue and county-level revenue, results were presented both for the base case (correlation=0.63) and a counterfactual case (correlation=0.9). The findings were that the optimal insurance choice under the AR design with a correlation of 0.9 and a scale factor of 1.5 (insured to planted acres) was to take the maximum allowable coverage of actuarially fair insurance. The reason for this is that at high correlations basis risk is lower than at low correlations, holding other factors constant. In other words, there is lower residual uninsurable risk and risk aversion dominates the precautionary demand for saving, causing the maximum allowable coverage to be taken at all wealth levels.

The impact of subsidized insurance on the farmer’s optimal choice was also investigated. The findings suggested that the maximum allowable coverage can be achieved at all initial wealth even for the case with relatively high basis risk. For the base case (correlation=0.63) a 20% subsidy would be sufficient to cause the maximum allowable coverage to be taken.

In conclusion, this study has shown that many of the standard results from static insurance models also hold in a dynamic model. It has also been shown that as long as complete credit markets exist and the farmer can borrow and save freely at going interest rates, consumption smoothing has no effect on insurance decision if there is no basis risk and insurance is moderately priced. Finally, it has also shown that if there is basis risk and/or a liquidity constraint exists then, consumption smoothing can have an impact on
the optimal insurance decision and, in some cases self-insurance will be preferred over formal insurance.

The model used in this study is obviously simplistic compared to actual situations faced by farmers. Therefore, while the findings of this research provide insights to economists who are concerned with issues of crop insurance, more work is required to translate these findings into outputs that farmers, lenders and policy makers can use directly. For example, no productive assets were considered in the model and, therefore, no capacity to expand operations (investment) exists in the model. Also, there is an aggregate store of wealth and that is implicitly a bond that can be traded short or long. The farmer could not buy stocks, or corporate bonds, or livestock, or any kind of productive assets. Furthermore, no government programs were considered and all revenue was assumed to be insurable. It was also assumed that premium payments were made when the insurance choice is made and cannot be deferred until harvest. These simplifying assumptions were necessary in order to keep the analysis of this research tractable for the present purpose.

Possible extension of this research is to calibrate the model to a more practical farmer situation which relaxes some of the simplifying assumptions above. In addition an empirical investigation of the performance of credit markets to determine if farmers are able to smooth their consumption efficiently, could potentially shed more light to the question of why there appears to be weak participation in U.S. crop insurance programs. In other words, the is a need to determine if and, to what extent, U.S. farmers face liquidity constraints. Further, an investigation of the level of basis risk in existing crop
insurance contracts will also provide useful insights. In this study, the source of basis risk considered was yield basis. A study that also considers price basis risk (together with yield basis risk) should be an interesting extension.
APPENDIX
Figure A1. Terminal value function

```c
#include "tv.h"

/*-----------------------------Revenue Insurance model optimization -- terminal value function.
 */

{ /*-----------------------------------------------*/
    result = 0;
    /*-----------------------------------------------*/
    return(result);
} 
```
#include "sd.h"

Insurance model optimization -- state dynamics function.

**/*
INT k = 353;  /* farm revenue guarantee */
DECIMAL rate = 0.0543;  /* interest rate */
load = 1;  /* insurance loading factor */
farm = 0;  /* initialize farm revenue */
prem = 0;  /* initialize insurance premium */
gross[12]={63, 111, 158, 206, 253, 301, 348, 396, 443, 491, 538, 586},
prob[12]={0.0002, 0.0015, 0.0093, 0.0384, 0.1067, 0.1995, 0.2509, 0.2123,
0.1208, 0.0463, 0.0119, 0.0022};

compute insurance premium

prem = prob[0]*(max(0, dec[0]*k-gross[0])) +
prob[1]*(max(0, dec[0]*k-gross[1])) +
prob[2]*(max(0, dec[0]*k-gross[2])) +
prob[3]*(max(0, dec[0]*k-gross[3])) +
prob[4]*(max(0, dec[0]*k-gross[4])) +
prob[5]*(max(0, dec[0]*k-gross[5])) +
prob[6]*(max(0, dec[0]*k-gross[6])) +
prob[7]*(max(0, dec[0]*k-gross[7])) +
prob[8]*(max(0, dec[0]*k-gross[8])) +
prob[9]*(max(0, dec[0]*k-gross[9])) +
prob[10]*(max(0, dec[0]*k-gross[10])) +
prob[11]*(max(0, dec[0]*k-gross[11]));

Implement state dynamics equations. i.e the transition equations

nxt_state[2] = 198.5919 + 0.4514*cur_state[2] + outcome[0];  /*county revenue*/
farm = -35.9214 + 1.0734*nxt_state[2] + outcome[2];  /*farm revenue*/
nxt_state[1] = 15.3369 + 0.9049*cur_state[1] + outcome[1];  /*production costs*/
nxt_state[0] = (1+rate)*(cur_state[0] - dec[1] - cur_state[1] - (1/(1+rate))*load*prem) + max(dec[0]*k, farm);  /*wealth*/

return;}

Figure A2. State dynamics function
#include "obj.h"

Insurance model optimization -- stage return function.

INT risk = 2, /* Risk aversion parameter */
    k = 353;   /* Gross revenue guarantee */

DECIMAL rate = 0.0543, /* interest rate */
    load = 1,   /* insurance loading factor */
    farm = 0,   /* initialize farm revenue */
    m = -1000,  /* min net wealth position */
    prem = 0,
    gross [12]={ same as in state dynamics function},
    prob [12]={ same as in state dynamics function};

 Compute current period reward

if (dec[1]==0)
    result = -1000;
else if ((cur_state[0]-cur_state[1]-dec[1]-(1/(1+rate))*load*prem)<m)
    result = -10000;
else /*
    result = (1/(1-risk))*pow(dec[1],(1-risk));
    */
    return(result);

Figure A.3. Stage Return Function
Figure A.4a. Scenario File, Part 1
DISTRIB "farm"; RV "farm";
EVENT  0.0002  -264.98  ;
EVENT  0.0015  -217.48  ;
EVENT  0.0093  -169.98  ;
EVENT  0.0384  -122.48  ;
EVENT  0.1067  -74.98   ;
EVENT  0.1995  -27.48   ;
EVENT  0.2509   20.02   ;
EVENT  0.2123  67.52    ;
EVENT  0.1208  115.02   ;
EVENT  0.0463  162.52   ;
EVENT  0.0119  210.02   ;
EVENT  0.0022  257.52   ;

! Discrete state-space grid

STAGE 10 0 0; COMBINE -1000|1000 160|160 362|362, 0.00|0.85|0.05 0|600|5, "revenue" "cost" "farm";
REFERENCES
REFERENCES


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