ABSTRACT

ESSAYS ON LAND USE DECISIONS FOR ENERGY CROP PRODUCTION AND THE EFFECTS OF SUBSIDIES UNDER UNCERTAINTY AND COSTLY REVERSIBILITY

By Feng Song

The U.S. Energy Independence and Security Act of 2007 mandates blending into transportation fuels of 16 billion gallons of cellulosic fuels annually by 2022. Dedicated energy crops are being explored to provide more efficient and environmental friendly feedstocks for cellulosic biofuel production. Devoting land to energy crops represents a long-term commitment, involves adjustment costs and great uncertainties. This research develops a dynamic land conversion model to take into account these factors. The model is applied to address two separate but related questions: under what conditions farmers are willing to convert production land to energy crops? Which subsidy policies encourage energy crop production most cost effectively?

This dissertation is divided into two essays. The first essay studies a farmer’s decision to convert a unit of traditional crop land into dedicated energy crops, taking into account sunk conversion costs, uncertainties of traditional and energy crop returns, and learning. The optimal decision rules differ significantly from the expected net present value rule, which ignores learning, and from real option models that allow only one way conversions into energy crops. These models also predict drastically different patterns of land conversions into and out of energy crops over time. Using corn-soybean rotation and switchgrass as examples, we show that the model predictions are sensitive to assumptions about stochastic processes of the returns.
The second essay evaluates the cost-effectiveness of four types of governmental subsidies in encouraging energy crop production. We first present a land conversion model to show how the subsidies that are expected net present value (ENPV) equivalent can change a representative farmer’s optimal land conversion rules differently for converting land into an alternative use as well as converting out of it. This is because these subsidies affect the land conversion costs, land return level and uncertainty differently. Then in the context of encouraging switchgrass production, we compare the probabilities of inducing the representative farmer to convert land from corn-soybean to switchgrass across four subsidies for the same, fixed 30-year expected government budget. Results of Monte Carlo simulations show that the insurance subsidy results in the highest probability of land being converted to the energy crop, followed by the constant subsidy. Although the cost-sharing subsidy and the variable subsidy encourage land conversion to the energy crop, they also discourage land from staying in it. Over time, these two subsidies have little effect on the land area in energy crops compared to the no-subsidy baseline. Combining the establishment cost-sharing subsidy with other annual subsidies has no added effect over single subsidies in inducing land conversion to the energy crop.
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Introduction

The soaring oil price between 2005 and 2008 coupled with pressures of energy security and climate change crisis have triggered another round of efforts to search for renewable energy sources. Included in the solution portfolio is biomass energy, which is especially attractive in countries with ample agricultural land, such as United States. In addition to co-firing to generate electricity, special interest is given to convert biomass to transportation fuel. Although currently most U.S. liquid biofuels are derived from corn grain, cellulosic biofuels are being strongly advocated because of their better environmental performance (Schemer et al. 2008; Paine et al. 1996). The U.S. Energy Independence and Security Act (EISA) of 2007 mandates the use of cellulosic biofuel, increasing from 0.1 billion gallons annually in 2010 to 16 billion gallons in 2022. Dedicated energy crops, such as switchgrass, have been identified as the one of the major feedstocks for cellulosic biofuel production in the United States (Perlack et al. 2005).

As pointed by Khanna (2008), to be economically viable, energy crops must compete successfully both as crops and as fuels. Cellulosic biofuel produced from the energy crops need to compete with fossil fuels and corn–based ethanol. Farmers need to make land allocation decisions for energy crops. Besides expected profitability considerations, which underpin the net present value (NPV) decision rule, uncertainty plays an important role in farmers’ adoption decision of energy crops since growing them is fraught with risks. Farmers face at least two categories of risks: policy risks and market risks. Given current technology, the potential demand for energy crops as cellulosic feedstock is largely from ethanol blending mandate of EISA. There are concerns about how the mandate is going to be implemented (Taheripour and Tyner 2008). More importantly,
although the price of energy crops is largely undetermined, it is likely to exhibit different volatility patterns from traditional crops prices. As ethanol feedstocks, energy crops will have prices determined in large part by the ethanol market, which is linked to the gasoline market (Tyner 2008). Energy crop price volatility is likely to be aggravated as ethanol shifts in and out of status as a cost-effective fuel substitute for gasoline, based on the relative prices of petroleum and corn-based ethanol. In short, the uncertainties should be taken into account in modeling farmers’ adoption decision of the energy crops. This dissertation focuses on the market risks.

Studying farmers’ land use decision mechanism also has important implications on design of effective governmental policies to support energy crop production. Coupled with energy policies that induce energy crop production by creating new markets for them, many countries also use agricultural subsidies to provide direct production incentives. These range from cost-sharing at start-up to price subsidy at final sale. The effectiveness of these subsidies should be systematically analyzed and can only be correctly evaluated if we can understand how the farmers make decisions.

The real options framework has been used to investigate the effects of uncertainty on the timing of land conversion to alternative uses. The general finding is that combined effects of uncertainty and sunk costs involved in the conversion generate substantial “hurdle rates”, which is not accounted for in present value analyses that are based on expected values. However it is often assumed the land conversion decision is irreversible. This may be a reasonable assumption for urban land conversion, but for agricultural land, a farmer can switch between different uses at some cost. Therefore we adopt an innovative approach by allowing the costly conversion of land back to original use. With
this improved model, I address two related questions about land use decisions for energy crop production. In the first essay I derive the dynamically optimal land conversion rule between a dedicated energy crop and a traditional food crop rotation. The resulting decision rule is contrasted with alternative decision rules, including the NPV rule and the one-way dynamically optimal conversion rule. We also compare two stochastic processes for payoffs from the two land uses using market and agronomic yield data. These processes go beyond the typical geometric Brownian motion assumptions in the literature. The second essay examines four types of ENPV-equivalent subsidies, showing how they can affect a representative farmer’s optimal land conversion rule differently, depending on their effects on the conversion costs, level of returns and variability of returns. For a given governmental budget, the subsidies are compared by their ability to attract land conversion to energy crops.
References


Essay 1
Switching to Perennial Energy Crops under Uncertainty and Costly Reversibility

Introduction

Replacing fossil fuel with renewable fuels, including biofuels such as ethanol, has been advocated for contributing to energy independence and mitigating climate change. Currently most of the ethanol produced in the United States comes from corn grain, raising concerns about the negative environmental impacts associated with corn production, upward pressure on food prices, and greenhouse gas emissions due to indirect land use changes as rising food prices induce cultivation of new lands (Searchinger et al. 2008). A promising alternative is cellulosic ethanol, which relies on nonfood feedstocks. The U.S. Energy Independence and Security Act (EISA) of 2007 mandates blending into transportation fuels of 36 billion gallons of renewable liquid fuels annually by 2022, out of which at least 16 billion gallons must be cellulosic ethanol. Significant expansion of cellulosic ethanol production will require more land to grow dedicated energy crops. A recent simulation of potential US switchgrass production implies a need for 71 million acres of crop land to meet the 2007 EISA mandate (Thomson et al. 2008). Yet idle land in the United States, including CRP land, is only about 40 million acres (Lubowski et al. 2006). This implies that current production land will need to be converted to grow cellulosic energy crops.

Although large scale production of cellulosic energy crops is not commercially viable at present, its advent could have dramatic effects on land use change and associated economic and environmental impacts. Forecasting the conditions under which such change would occur is an important first step toward evaluating likely outcomes and relevant policy interventions. Forecasting land use change depends critically on farmers’ land use decisions, which in turn are driven by several salient features of dedicated energy crops.

1 A manuscript Song, Zhao and Swinton (2010), which is based on the content of this essay, has been submitted for consideration for publication.
First, all of the major cellulosic energy crop contenders are perennial. They need several years to establish before achieving full yield potential (Powlson, Riche, and Shield 2005). Devoting land to energy crops represents a long-term commitment by the farmer and incurs sunk costs. Moreover, converting land back to traditional annual crops also incurs (possibly substantial) costs (e.g., costs of killing persistent perennial rootstocks).

Second, farmers growing cellulosic energy crops face great revenue uncertainty due to both production and price uncertainties. The production uncertainties are inherent in any agricultural production. More importantly, energy crop prices are largely undetermined but likely to exhibit different volatility patterns from traditional crops. Crops destined for conversion into ethanol will have prices determined in large part by the ethanol market, which is linked to the gasoline market (Tyner 2008). Energy crop price volatility is likely to be aggravated as ethanol shifts in and out of status as a cost-effective fuel substitute for gasoline, based on the relative prices of petroleum and corn grain, the leading current ethanol feedstock in the United States. Although mandated growth in cellulosic ethanol demand under EISA may mitigate one policy related source of price uncertainty, there remain important uncertainties regarding federal climate-change policy and state-level renewable energy policies.

Finally, although real options methods exist for modeling stochastic revenue streams and uncovering optimal decision rules, the real options literature typically relies upon Brownian motion that is mathematically convenient but not necessarily consistent with observed variability of energy returns. For instance, Pindyck (1999) examined the long-run evolution of oil, coal and natural gas prices using the U.S. data from year 1870 to 1996 and found that these energy prices are mean reverting, in spite that the rate of reversion is slow. In capital investment literature, the effect of different assumption on the underlying stochastic process has been investigated by

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2 Another large demand for energy crops is co-firing to provide electricity.
Metcalf and Hassett (1995) and Sarkar (2003), etc. Some found that the effect is negligible while others gave opposite conclusion. It is of interest to examine how the assumption of different stochastic processes will affect the agricultural land conversion decision.

In this paper, we study land conversions between traditional crops and energy crops, incorporating these three features of energy crops. We make several contributions to the literature. First, we extend studies based on the net present value (NPV) approaches to allow for uncertainty, sunk costs and learning. In the NPV approach, a farmer will convert land to energy crops if the expected NPV of the returns from energy crops exceed those from current (traditional) crops. But under uncertainty and sunk costs, the farmer may be more reluctant to convert land into and out of the two uses, similar to the predictions of real option theory (Dixit and Pindyck 1994).

Second, we extend the real options studies and allow for land use conversion in two directions, so a farmer deciding on converting to energy crops is allowed to take into consideration the future possibility of converting the land back to traditional crops under plausible market conditions. Real option theory has been widely applied in urban land use decisions since Titman (1985) (e.g., Capozza and Li 1994; Abebayehu, Keith, and Betsey 1999). A common assumption in this literature is that land conversion is irreversible. This assumption might be reasonable for urban development, but for agricultural land, a farmer can switch between different uses with costs. Allowing two-way land conversion is important to capture the flexibility of farmer’s land use decisions, and is particularly important for energy crops given the high degrees of uncertainties involved.

Third, we contrast the effects of two alternative stochastic processes for returns from the two competing crop choices, geometric Brownian motion (GBM) and mean reverting (MR). We use

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3 Here we model an autonomous learning process. The uncertainties of the crop returns change over time. By observing the historical realizations of crop returns, the farmer can update his information set and obtain the new marginal distribution of the returns in the future.
both historical market data and simulated agronomic yield data to parameterize and test the stochastic processes and show how optimizing farmer behavior responds to alternative assumptions about the stochastic processes.

Our paper is closely related to the broader real options literature (Dixit and Pindyck 1994), especially those allowing for two-way decisions. Dixit (1989) studies a firm’s entry and exit decisions assuming that the output price is the only state variable and evolves according to GBM. Mason (2001) extends this work to examine a mine’s decision to start and to shut down, for which not only the output price is uncertain but the resource reserve is limited. This results in the optimal decision rule depending on both the reserve stock level and the price. Dixit (1989) and Mason (2001) both assume that the decision maker only obtains a return in one state (entry or active) and thus only one stochastic state variable (price) governs the optimal action. Our model allows two separate but possibly correlated returns, those from traditional crops and from energy crops. Kassar and Lasserre (2004) examine the optimal abandonment rule between two species, both of which have stochastic values. However, in their study, a species cannot be recovered once it is lost, which is equivalent to switching in one direction only.

A General Land Conversion Model

Consider a risk-neutral farmer\(^4\) with a unit of land facing two land use alternatives, \( i \in S = \{c, s\} \), who can convert from alternative \( i \) to \( j \) with a lump-sum sunk cost \( C_{ij} \). For concreteness, we use corn-soybean rotation (denoted by \( c \)) and switchgrass (denoted by \( s \)) as representative of a traditional crop and an energy crop. The return to alternative \( i \) in period \( t \) is denoted by \( \pi_i(t) \).

\(^4\) We assume risk neutrality for simplicity; similar results can be obtained when the farmer is risk averse.
which is assumed to evolve according to a known stochastic process of the general form:

\[ d\pi_i(t) = \alpha_i(\pi_i,t)dt + \sigma_i(\pi_i,t)dz_i \quad i = c, s \]

where the drift term \( \alpha_i(\pi_i,t) \) and the variance term \( \sigma_i(\pi_i,t) \) are known nonrandom functions, and \( dz_i \) is the increment of a Wiener process. Thus, new information about future returns of the two crops comes in the form of newly observed return levels, which become starting points for the distributions of future returns. Note that we model the returns directly, instead of modeling the price and yield uncertainties separately. This approach simplifies our analysis, and in the empirical section we derive the return processes from the underlying price, yield and cost processes. The correlation coefficient of the two return processes is \( \rho \), i.e., \( E(dz_cdz_s) = \rho dt \).

Traditional crop and energy crop returns could be correlated for a variety of reasons, e.g. both are linked with energy prices and are subject to macro-economic shocks. Moreover, farmers switching land between the two crops could introduce a long-term relationship between their return series. Finally, let \( r \) be the farmer’s discount rate.

A key insight of the real options approach is that when the land is in use \( i \), say in traditional crops, the farmer has the option of converting it into energy crops when market conditions are “favorable.” Once converted, it is costly to revert it back to traditional crops if the market conditions turn out to be less favorable. Thus, sticking to the current land use (in traditional crops) has an additional value, called option value, derived from the option of converting it into the alternative use (in energy crops). But since the land in energy crops can be further converted back to traditional crops (albeit at a cost), this option value of converting from traditional to energy crops further depends on the option value associated with converting from energy to traditional crops. The mutual dependence of the two option values significantly complicates the solution algorithm. To make the analysis traceable and rigorous, we solve a simplified model in Appendix.
A, which admits an analytical solution. Thus we can reveal some useful information through comparative statics.

Let \( V^i_j(t, \pi_c(t), \pi_s(t)) \) be the farmer’s period \( t \) expected present-value payoff starting with land use \( i \) and following optimal land conversion rules. Due to the option of converting into use \( j \neq i \), the payoff depends on the distribution of future returns of both land uses, the information for which is contained in the two current returns, \( \pi_c(t) \) and \( \pi_s(t) \). At time \( t \), the farmer chooses between keeping the land in use \( i \) and converting it into alternative use \( j \):

\[
(2) \quad V^i_j(t, \pi_c(t), \pi_s(t)) = \max \left\{ \pi_i(t)dt + e^{-rdt} E V^i_j(t + dt), \pi_s(t + dt), V^j_j(t, \pi_c(t), \pi_s(t)) - C_{ij} \right\}
\]

The first term on the right-hand side describes the payoffs if the land is kept in use \( i \): in the infinitesimal period \([t, t + dt]\), the farmer receives profit from land use \( i \) at rate \( \pi_i(t) \), and at the end of the period, receives the new discounted expected payoff \( e^{-rdt} E V^i_j(t + dt) \). The second term on the right-hand side describes the payoff if the land is converted into use \( j \): the farmer receives the expected payoff of use \( j \), \( V^j_j(t) \), but incurs the conversion cost \( C_{ij} \).

As shown by Brekke and Oksendal (1994), the decision problem in (2) can be equivalently expressed by a set of complementary slackness conditions, as long as the value functions \( V^c(\bullet) \) and \( V^s(\bullet) \) are stochastically \( C^2 \).\(^5\) First, (2) implies that

\(^5\) \( V^i(\bullet) \) is stochastically \( C^2 \) if it is continuously twice differentiable in \( \pi_i \) and satisfies the generalized Dynkin formula, which gives the expected value of any suitably smooth statistic of an Itô diffusion process at a stopping time. See more details in Brekke and Oksendal (1994). This condition is trivially satisfied in our applications.
which, after applying Ito’s lemma, can be expressed as \[ LV^i(\pi_c(t), \pi_s(t)) \geq 0, \]
where

\[
LV^i(\pi_c, \pi_s) = rV^i(\pi_c, \pi_s) - \pi_i(t) - \alpha_c(\pi_c, t)V^i_{\pi_c} - \alpha_s(\pi_s, t)V^i_{\pi_s}
- 1/2\sigma_c^2(\pi_c, t)V^i_{\pi_c\pi_c}
- 1/2\sigma_s^2(\pi_s, t)V^i_{\pi_s\pi_s}
- \rho\sigma_c(\pi_c, t)\sigma_s(\pi_s, t)V^i_{\pi_c\pi_s},
\]
and the subscripts of \(V^i\) denote partial derivatives.\(^6\) Equation (2) also implies that

\[ V^i(\pi_c(t), \pi_s(t)) \geq V^j(\pi_c(t), \pi_s(t)) - C_{ij}. \]

Then we know the value functions \(V^i\) and \(V^j\) have to satisfy:

1. \[ LV^i(\pi_c, \pi_s) \geq 0, \quad i = c, s \]
2. \[ V^i(\pi_c, \pi_s) \geq V^j(\pi_c, \pi_s) - C_{ij}, \quad i, j \in \{c, s\} \text{ and } i \neq j \]
3. either (i) or (ii) has to hold as a strict equality.

If (i) is an equality, the farmer should keep his land in current use \(i\), and if (ii) is an equality, the farmer should switch the land use to \(j\). If both are equalities (a nongeneric case), the farmer is indifferent between converting and not converting. The optimal conversion decisions (i.e.,

\[ e^{-r*dt}EV^i(\pi_c(t + dt), \pi_s(t + dt)) \]

and

\[
\frac{1}{1 + r*dt}[V^i(\pi_c, \pi_s) + EdV^i], \quad \text{which, by applying Ito's lemma, can be written as}
\frac{1}{1 + r*dt}[V^i(\pi_c) + (\alpha_c(\pi_c, t)V^i_{\pi_c} + \alpha_s(\pi_s, t)V^i_{\pi_s}) + 1/2\sigma_c^2(\pi_c, t)V^i_{\pi_c\pi_c}
+ 1/2\sigma_s^2(\pi_s, t)V^i_{\pi_s\pi_s}
+ \rho\sigma_c(\pi_c, t)\sigma_s(\pi_s, t)V^i_{\pi_c\pi_s})dt]. \]

After multiply both sides of equation by \((1 + r*dt)\), rearrange the terms and divide both sides by \(dt\), we can obtain equation (4).
solutions to (5)) are represented by two conversion boundaries in the $\pi_c - \pi_s$ space, one for each type of current land use, as shown in figure 1.1. If the current land use is in traditional crops ($i=c$), the conversion boundary, $\pi_s = b^{cs}(\pi_c)$, denotes the returns from traditional and energy crops that the farmer is indifferent between converting to energy crops and sticking to traditional crops. Above this boundary, i.e., when $\pi_s > b^{cs}(\pi_c)$, the returns from energy crops are sufficiently high that it is optimal for the farmer to convert to energy crops. Conversely, if $\pi_s < b^{cs}(\pi_c)$, the farmer should stick to growing traditional crops. Intuitively, as shown in figure 1.1, boundary $b^{cs}$ lies above the 45° line; given the sunk costs and uncertainties, the farmer is reluctant to convert to energy crops even when its return $\pi_s(t)$ slightly exceeds $\pi_c(t)$, the return from traditional crops.

Similarly, if the current land use is in energy crops, the boundary for converting to traditional crops is given by $\pi_s = b^{sc}(\pi_c)$. If the return from energy crops is too low compared with traditional crops so that $\pi_s < b^{sc}(\pi_c)$, the farmer should convert to traditional crops. Otherwise, it is optimal for the farmer to stick to the current use (in energy crops). Again, due to uncertainty and sunk costs, $b^{sc}$ lies below the 45° line in 1.1: once the land is already in energy crops, the farmer is reluctant to convert it to traditional crops unless the return from traditional crops is sufficiently high. Thus, the two boundaries divide the $\pi_c - \pi_s$ space into three regions: above the boundary $b^{cs}$, it is optimal to convert from traditional crops to energy crops; below the boundary $b^{sc}$, it is optimal to convert from energy crops to traditional crops; and between the two boundaries, it is optimal to keep land in its current use.
Since the two value functions \( V^C(\cdot) \) and \( V^S(\cdot) \) are interdependent, there are no analytical solutions to (5). We follow Miranda and Fackler (2002) and employ the collocation method, which approximates the unknown value functions \( V^i(\pi_c, \pi_s) \) using a linear combination of \( n \) known basis functions:

\[
\hat{V}^i(\pi_c, \pi_s) = \sum_{j_C=1}^{n_C} \sum_{j_S=1}^{n_S} c_{j_C,j_S} \phi_{j_C,j_S}(\pi_c, \pi_s)
\]

Where \( \hat{V}^i(\cdot) \) represents the numerical approximation of \( V^i(\cdot) \). Compared to the shooting algorithm and finite difference method that are often used to numerically solve the value functions in stochastic dynamic optimization problems, the collocation method is a fast and robust alternative (Dangl and Wirl 2004). The coefficients \( c_{j_C,j_S} \) are found by requiring the approximant to satisfy the optimality condition (5) at a set of interpolation nodal points. The threshold returns that induce the farmer to convert land will also be solved at the nodal points. The details about the numerical solution are documented in Appendix B.

**Data and Parameter Estimation**

The empirical analysis focuses on a representative farmer’s optimal land conversion decision in the North-central United States, where corn and soybean are two major crops frequently grown in rotation. We assume that the farmer currently grows corn and soybean in a balanced rotation.

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7 If we allow land conversion in one direction only, then analytical solutions exist for special stochastic processes of \( \pi_c(t) \) and \( \pi_s(t) \), e.g., geometric Brownian motions. Numerical methods must be employed for more general processes even for one-way conversion models. For instance, Insley and Rollins (2005) and Conrad and Kotani (2005) formulate (4) as a linear complementarity problem and then use the finite difference approach to solve it numerically.

8 The North-central area includes Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Ohio, and Wisconsin.
(with half of his land in each crop). The alternative land use is to grow switchgrass. The model can be easily adapted to other locations and crops.

**Construction of Returns to Land Uses**

We first estimate the drift and variance parameter functions \( \alpha_i(\pi_i, t) \) and \( \sigma_i(\pi_i, t) \) for the two land uses, and the correlation coefficient of the two stochastic processes \( \rho \). Typically, these parameters are estimated using historical time series on returns. We use U.S. Department of Agriculture’s North-central regional data for 1982 to 2008 (USDA 2008) to calculate the annual average return to the corn-soybean rotation, which equal the value of production minus the operating cost. The returns are deflated by the Consumer Price Index (CPI, 1982=100). The data series is plotted in figure 1.2.

Since switchgrass has not been grown commercially as a biofuel feedstock, we do not have historical data for switchgrass returns. We instead construct a hypothetical series of returns for switchgrass grown as an energy crop. The return equals the farmgate price of switchgrass times yield minus the production costs. The farmgate price is determined by ethanol producers’ willingness to pay (WTP) as well as government subsidies. Ethanol producers’ WTP is equal to the ethanol price minus the conversion costs from switchgrass to ethanol and the transportation costs from field to a processing facility. The ethanol price (in $ per gallon) is obtained from the Nebraska Energy Office from 1982 to 2008. The estimated conversion cost is assumed to be $0.91 per gallon (DiPardo 2004). Assuming that one ton of switchgrass yields 91 gallons of ethanol (Schmer et al. 2008) and multiplying by this conversion rate, we convert the ethanol price and conversion cost from a per gallon basis into per ton basis. The transportation cost is assumed to be $8 per ton (Babcock et al. 2007). For government subsidies, we use the $45/ton matching

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9 Governmental subsidies are important in the competitiveness of switchgrass. With our baseline parameters assumptions, ethanol prices in some years were so low that the switchgrass revenue was not able to cover the production costs if there had been no governmental subsidy.
payment currently provided by USDA to biofuel producers for their costs of collection, harvest, transport and storage of biomass. Switchgrass yield for 1982-2008 in the same North-central states as the corn-soybean returns data is obtained from Thomson et al. (2008), who used the EPIC Model at the 8-digit watershed level that can be aggregated to the state level. The average switchgrass yield is 3.10 tons/acre and the standard deviation is 0.20 tons/acre. The production cost is assumed to be $190/acre (Duffy and Nanhou 2001). This is the operation cost in maintenance year 2-10, which does not include the establishment cost since it will be accounted in the land conversion cost discussed below. Finally, the nominal returns are deflated to a 1982 base using the CPI. These returns are plotted in figure 1.2.

Parameter Estimation

We consider two commonly used return processes, geometric Brownian motion (GBM) and geometric mean reversion (MR). The GBM process is widely used in real option studies for its analytical tractability, and is represented as

\[ d\pi_i = \alpha_i \pi_i dt + \sigma_i \pi_i dz_i, \quad i = c, s \]

where \( \alpha_i \) and \( \sigma_i \) are drift and variance parameters respectively. If a return follows a GBM process, the conditional mean and variance of the return rise over time without boundary. Thus it is a nonstationary series. The MR process, on the other hand, assumes that the random variable will revert to a long-term average, is stationary and is described by

\[ d\pi_i = \eta_i (\pi_i - \bar{\pi}_i) \pi_i dt + \sigma_i \pi_i dz_i, \quad i = c, s \]

where \( \bar{\pi}_i \) is the long-term average return of land use \( i \), \( \eta_i \) is the speed of reversion and \( \sigma_i \) is the variance rate. The returns revert to a long-run equilibrium \( \bar{\pi}_i \) at a speed of \( \eta_i \bar{\pi}_i \). The further the returns divert from the \( \bar{\pi}_i \), the quicker the reversion will be.
Theoretically, both processes can be justified in describing how agricultural returns evolve over
time. GBM can better reflect a trend which could be positive due to technological advances that
boost productivity while a MR process can better reflect the long-term equilibrium conditions
when technology is unchanging. Statistical tests of the stationarity of corn-soybean and
switchgrass returns generate mixed signals. For instance, for the logarithmic corn-soybean returns,
the null hypothesis of unit root can not be rejected at 10% level based on the Dickey-Fuller test
and Phillip-Perron test, but the null hypothesis of stationarity can not be rejected at 10% level
based on KPSS test, either. Similar results are found for logarithmic switchgrass returns. The
detailed test results can be found in Appendix C. Given the nascent state of biofuel crop
technology and markets, the dynamic growth implicit in a GBM process seems the more justified
of the two. Thus, we use GBM processes in our baseline model, and study how the results change
if MR processes are used instead.

To estimate the parameters of the two processes for corn-soybean and switchgrass returns, we
first discretize the two continuous time processes. If $\pi_i$ follows a GBM, then $\ln \pi_i$ follows a
simple Brownian motion with drift, which is the limit of a random walk. In particular,

$$\ln \pi_{i,t+1} - \ln \pi_{i,t} = (\alpha_i - 1/2\sigma_i^2) + \sigma_i \varepsilon_i, \quad i = c, s$$

where $\varepsilon_i \sim N(0,1)$. The maximum likelihood estimates of the drift $\alpha_i$ and the standard
deviation $\sigma_i$ are thus $\hat{\alpha}_i = m_i + 0.5s_i^2$ and $\hat{\sigma}_i = s_i$, where $m_i$ and $s_i$ are respectively the
mean and standard deviation of the series $\ln \pi_{i,t} - \ln \pi_{i,t-1}$. The estimate of the correlation
coefficient $\rho$ is the correlation between series $\ln \pi_{c,t} - \ln \pi_{c,t-1}$ and $\ln \pi_{s,t} - \ln \pi_{s,t-1}$.
The parameter estimates for the GBM representation of the corn-soybean and switchgrass returns are presented in table 1.\(^\text{10}\)

The discrete time approximation to the MR process in (8) is as follows:

\[
\pi_{i,t} - \pi_{i,t-1} = \eta_i (\pi_{i,t} - \pi_{i,t-1}) + \sigma_i \pi_{i,t-1} e_i, \quad i = c, s
\]

where again \(e_i \sim N(0,1)\). Dividing both sides by \(\pi_{i,t-1}\), we obtain

\[
\frac{\pi_{i,t} - \pi_{i,t-1}}{\pi_{i,t-1}} = a_i + b_i \pi_{i,t-1} + \sigma_i e_i, \quad i = c, s
\]

Since the stochastic processes of the returns to corn-soybean and switchgrass have correlated residuals (i.e., \(e_c\) and \(e_s\) are correlated), we use a seemingly unrelated regression model to estimate the parameters \(a_i\), \(b_i\), \(\sigma_i\) and \(\rho\). Consistent estimates of \(\eta_i\) and \(\pi_i\) are then obtained as \(\hat{\eta}_i = -\hat{b}_i\), and \(\hat{\pi}_i = -\frac{\hat{a}_i}{\hat{b}_i}\). The regression estimates are as follows: \(\hat{a}_c = 0.48\), \(\hat{b}_c = -0.0046\) with standard errors 0.20 and 0.0019, respectively, and \(\hat{a}_s = 0.72\), \(\hat{b}_s = -0.0038\) with standard errors 0.35 and 0.0022, respectively. The standard error of the regression is 0.31 and 1.02 for corn-soybean and switchgrass respectively. The calculated results for \(\eta_i\) and \(\pi_i\) are reported in table 1. Consistent with the observation in figure 1.2, the return to switchgrass has higher volatility than that to corn-soybean, no matter which type of stochastic processes they follow.

\(^{10}\) The estimated growth rate of the switchgrass return is 0.17, which is higher than the assumed discount rate of 0.08. For the dynamic optimization problem to have a solution, the expected growth rate cannot exceed the discount rate (otherwise, the expected payoff from switchgrass is infinite). We thus assume that the switchgrass return grows at the same rate as corn-soybean. We later tested the results sensitivity to this assumption by varying the growth rate of switchgrass return from 0.01 to 0.07.
Land conversion between traditional crops and energy crops will introduce an equilibrium relationship between the two markets. Thus these two crop return series could be cointegrated. However, using the constructed historical data may fail to capture this relationship since the agricultural market and energy market were not integrated historically (Tyner and Taheripour 2008; Tyner 2009). Historical returns to corn-soybean and switchgrass could be negatively correlated as indicated by our estimation results (-0.24 under GBM assumption and -0.23 under MR assumption). Part of the reason is that switchgrass revenue is simulated as a function of petroleum price and thus highly positively correlated with petroleum, whereas until 2005, corn-soybean returns were negatively correlated with petroleum prices due to petroleum used as transportation and fertilizer inputs. However, this pattern of negative correlation could change as more corn and soybean are used to produce biofuels, and as agricultural and petroleum markets become more integrated. Then high petroleum prices may push up corn and soybean prices, increasing their returns. A supporting evidence is that the correlation between the annual ethanol price and corn-soybean return for year 1982-2005 is -0.07, and it changes to 0.28 for year 2006-2008. Tyner (2009) shows similar result that the price correlation between crude oil and corn change from -0.29 during period 1988-2005 to 0.8 during period 2006-2008. Furthermore, the positive correlation may become stronger as switchgrass or other energy crops expand production and compete with corn-soybean for limited land. In response to this possibility, we test the sensitivity of our results to the effect of positive correlation between the two crops.

*Land Conversion Costs*

As a perennial crop, switchgrass needs to become established and will not achieve full yield until the third or fourth year after seeding. It needs to be replanted every ten years. We use the NPV of the (infinite) sequence of first-year switchgrass establishment costs as an estimate of land
conversion costs from corn-soybean to switchgrass, $C_{cs}$.\textsuperscript{11} Switchgrass establishment costs include seed, chemicals, machinery and labor (e.g. Hallam, Anderson, and Buxton 2001; Duffy and Nanhou 2001; Khanna, Dhungana, and Clifton-Brown 2008; Perrin et al. 2008). The estimated costs vary widely across studies because of different assumptions, methods employed, and production locations. We use $109/acre estimated by Khanna, Dhungana, and Clifton-Brown (2008) because they report the detailed costs categories year by year, which facilitates our calculations. The NPV of the cost sequence is $136/acre at a discount rate of 8%.

Conversion of land from switchgrass to corn-soybean production requires clearing existing vegetation residue by tillage or herbicides. We use the costs of converting land in Conservation Reserve Program back to crop production as an approximate of the conversion costs from switchgrass to corn-soybean. Higher than normal fertilizer rates may be required for two years after conversion (Blocksome et al. 2008). We assume $47/acre conversion cost from switchgrass to corn-soybean production, which includes $17/acre disking operation costs and $30/acre total additional fertilizer costs for the first two years (Williams et al. 2009).

Results and Sensitivity Analysis

Given the baseline parameter values, we solve the optimality condition in (5) using OSSOLVER (Fackler 2004), implemented with CompEcon Toolbox in Matlab (Miranda and Fackler 2002). The same solver was employed by Nøtbakken (2006) to solve her model of a fleet’s optimal decision to enter or to leave a fishery. The family basis function we use is a piecewise linear spline. For each state variable (i.e., $\pi_c$ and $\pi_s$), the nodal points are evenly spaced over the revenue interval $[0, 5]$ (in hundred dollars) with an increment of 0.1.

\textsuperscript{11}The NPV of the decennial establishment cost for switchgrass ($136/acre) overestimates $C_{cs}$ if the farmer does not permanently stay with switchgrass. In this case, we overestimate the farmer’s reluctance to convert to switchgrass but not to a large extent (one time establishment cost is $109/acre).
Figure 1.3a shows the two boundaries (the solid lines) for conversions from corn-soybean to switchgrass ($b^{CS}$) and from switchgrass to corn-soybean ($b^{SC}$) assuming that both returns follow GBM. The boundaries indicate significant hysteresis in land conversion decisions. For instance, the real average annual returns based on 2008 prices in 1982 dollars are $\pi_c = $92/acre for corn-soybean and $\pi_s = $135/acre for switchgrass. If the land is currently in corn-soybean, the minimum switchgrass return for converting the land to switchgrass is $b^{CS}(92) = $295/acre, which is significantly higher than $135/acre. Thus, the land will be kept in corn-soybean rotation even though $\pi_s > \pi_c$. Conversely, if the land is already in switchgrass, the required minimum corn-soybean return for converting into corn-soybean is about $280/acre. Thus, the land will not be converted either.

Given the two boundaries, we calculate the expected probabilities that a piece of land in corn-soybean will be in switchgrass for each year during a 30 year period, with the 2008 returns as the initial (time zero) returns: $\pi_c (0) = $92/acre and $\pi_s (0) = $135/acre. Given this starting point, we draw $N (=5000)$ sample paths of the joint return processes for 30 years according to (7) and the parameter values in table 1. Each sample path of the two returns, 

$\{(\pi_c(t), \pi_s(t)), t = 1, \ldots, 30\}$, is then compared with the conversion boundaries, 

$(b^{CS}(\bullet), b^{SC}(\bullet))$, to decide whether the land is kept in its current use or should be converted to the alternative use. For instance, in year 1, when the land is still in corn-soybean, the realized returns on a particular sample path, $(\pi_c(1), \pi_s(1))$, are compared with boundary $b^{CS}$. If the realized returns are in the “no action zone” (e.g., if $\pi_s(1) < b^{CS}(\pi_c(1))$) according to the

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12 The two correlated stochastic processes $\{(\pi_c(t), \pi_s(t)), t \in [0,30]\}$ are approximated by the Euler method and implemented using Matlab’s Econometric toolbox.
optimal decision rule), the land is kept in corn-soybean, and similar comparisons are made in year 2. If, on the other hand, the realized returns are in the “conversion zone” (i.e., if
\[ \pi_s(1) \geq b^{cs}(\pi_c(1)) \]
the land is converted to switchgrass, and in year two, the realized returns
\[ (\pi_c(2), \pi_s(2)) \]
will be compared with boundary \( b^{sc} \) to decide whether the land should be converted into corn-soybean. Finally, for each period we count the number of sample paths on which the land is in switchgrass. Dividing this number by \( N \), we obtain the proportion of land in switchgrass for each period.

Figure 1.3b illustrates the proportion of land in switchgrass for the 30 year period (solid line). Since the starting level of switchgrass return (at $135/acre) is much higher than that of corn-soybean (at $92/acre), more land is gradually converted into switchgrass, peaking at 29% of the total land area. However, the switchgrass return also has a higher level of uncertainty, and eventually some land in switchgrass is converted back to corn-soybean, stabilizing at about 16% of the land area.

Comparison with NPV and One-Way Conversion Rules

We next compare the two conversion boundaries found above with those based on the NPV rule. According to the NPV decision rule, the farmer will switch from corn-soybean to switchgrass when the expected NPV of switching is higher than staying in corn-soybean, i.e., when
\[ E \int_0^\infty \pi_s(t) e^{-rt} dt - C_{cs} \geq E \int_0^\infty \pi_c(t) e^{-rt} dt . \]
Similarly, the farmer will convert from switchgrass to corn-soybean when
\[ E \int_0^\infty \pi_c(t) e^{-rt} dt - C_{sc} \geq E \int_0^\infty \pi_s(t) e^{-rt} dt . \]
Given that both \( \pi_c(t) \) and \( \pi_s(t) \) follow GBM, we use (7) and obtain two NPV conversion boundaries:
\[ b_{NPV}^{cs}(\pi_c) = \pi_c \frac{r - \alpha_s}{r - \alpha_c} + (r - \alpha_s)C_{cs} \] for conversion from corn-soybean to
switchgrass, and \( b_{NPV}^{sc} (\pi_c) = \pi_c \frac{r - \alpha_s}{r - \alpha_c} - (r - \alpha_s)C_{sc} \) for conversion from switchgrass to corn-soybean. The two NPV boundaries (in dash lines, based on GBM process) are shown in figure 1.3a.

As illustrated in figure 1.3a, the NPV rule predicts that the farmer will convert between land uses far more readily than under the dynamically optimal real options rule. For instance, if the corn-soybean return is $98/acre, the average historical return during 1975 - 2007, the farmer who grows corn-soybean will convert to switchgrass if the switchgrass return exceeds $105/acre, and the farmer who grows switchgrass will convert to corn-soybean if the switchgrass return is less than $100/acre. But the real option rule, given by \( b^{cs} (\pi_c) \) and \( b^{sc} (\pi_c) \), indicates that the corn-soybean farmer will convert to switchgrass only if the switchgrass return exceeds $315/acre, which is 3 times the NPV threshold. The switchgrass farmer will convert to corn-soybean only if the switchgrass return is lower than $40/acre, 60% lower than the corresponding NPV threshold.

The different conversion boundaries under these two decision rules, \( (b^{cs} (\pi_c), b^{sc} (\pi_c)) \) and \( (b_{NPV}^{cs} (\pi_c), b_{NPV}^{sc} (\pi_c)) \), imply different amounts of land converted between the two uses. Figure 1.3b compares the proportions of land in switchgrass under the real option and NPV rules. The NPV rule predicts that land will be quickly converted into switchgrass (peaking at 73% of total land area), followed by a gradual decline, and eventually stabilizing at about 57%. The predicted proportion of land in switchgrass is consistently higher than the predictions of the dynamically optimal model.
A real options model that only allows one-way conversion will predict significantly greater farmer reluctance to convert than a two-way conversion model, as shown in figure 1.4a. For instance, the threshold return for converting from corn-soybean to switchgrass, $b_{OW}^{CS}$, doubles $b^{CS}$, the threshold boundary when two-way conversion is accounted for. Similarly, the corn-soybean return threshold for a farmer to convert from switchgrass to corn-soybean is twice as high under the one-way conversion model compared to the two-way conversion model. Because of the increased hysteresis, the one-way real options model predicts much lower proportions of land in switchgrass, as shown in figure 1.4b.

**Effects of Different Stochastic Processes**

We next investigate the effects of assuming different stochastic processes by comparing the conversion boundaries and switchgrass proportion under GBM and MR processes. We first “anchor” the two processes so that they are comparable by estimating the parameter values of the two processes using the same time series data for $\pi_c$ and $\pi_s$. Estimates for the two processes are presented in table 1.1. This anchoring approach implies that the parameter values may not be completely comparable. For instance, although the variance rate for the corn-soybean return under the GBM assumption (at 0.29) is roughly the same as that under the MR assumption (at 0.31), the variance rate for switchgrass return under the GBM assumption (at 0.64) is estimated to be much smaller than that under the MR assumption (at 1.0).

The two-way conversion boundaries for corn-soybean and switchgrass returns follow distinct patterns according to whether the underlying stochastic processes follow GBM or MR parameters, as illustrated in figure 1.5a. The solid lines define the optimal land conversion boundaries assuming GBM. Under a GBM process, the conversion pattern mainly depends on the relative

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13 The two corresponding conversion boundaries, $b_{OW}^{CS}$ and $b_{OW}^{SC}$, are obtained by imposing a prohibitively large cost of reverting back the earlier conversion.
volatility and conversion costs. This yields a fairly symmetric boundary pair, albeit with a lower threshold for conversion from switchgrass to corn-soybean at low returns than vice-versa. Under a MR process, the pattern is asymmetric, with a lower threshold for conversion to switchgrass and a much higher threshold to corn-soybean for low return rate than under GBM. But this pattern at low return levels is reversed at higher returns, with declining tendency to convert to switchgrass and rising tendency to convert to corn-soybean. The difference in conversion boundary patterns arises from three distinct effects of MR as compared to GBM processes: the relative effects of uncertainty, distant time horizon and mean reversion.

The uncertainty effect follows from the higher relative volatility of the switchgrass return in MR process than in GBM case (3 vs. 2 times the corresponding corn-soybean uncertainty). Hence, the optimizing farmer is more reluctant to convert land into switchgrass as well as out of it.\(^\text{14}\) Due to the higher relative volatility for MR, the uncertainty effect raises the conversion boundary from corn-soybean to switchgrass and lowers the boundaries in both directions.

The distant time horizon effect arises from the higher projected long-term average return from switchgrass as compared to corn-soybean. This effect lowers the conversion boundary from corn-soybean to switchgrass and raises the conversion boundary from switchgrass to corn-soybean compared to GBM case.

While the previous two effects hold true for low as well as high return levels, the mean reversion effect on the conversion boundaries behaves differently at low return levels than at high ones. When both corn-soybean and switchgrass returns are high, mean reversion pulls them

\(^{14}\) The reluctance to convert out of switchgrass as the switchgrass return becomes more uncertain is a feature of the real option argument: as \(\sigma_S\) increases, it is more likely that future return \(\pi_S\) is high, which implies that the farmer should not convert out of switchgrass. In response, the farmer has more incentive to wait until \(\pi_S\) is low relative to \(\pi_C\) before converting out. This prediction is opposite to that of standard risk aversion arguments, and has been used by Schatzki (2003) to test real option versus risk aversion assumptions.
downward towards the long-term average. However, the switchgrass return reverts to its mean more slowly than the corn-soybean return because it has both a smaller reversion speed parameter and a smaller absolute difference between the current return and the long-term average. At high return levels, these effects lower the conversion boundary from corn-soybean to switchgrass and raise the boundary from switchgrass to corn-soybean. By contrast, when the corn-soybean and switchgrass returns are low, mean reversion causes them to rise. If the switchgrass return reverts more slowly than the corn-soybean return, it raises the boundary from corn-soybean to switchgrass and lowers the boundary from switchgrass to corn-soybean. When the order of the reversion speed is reversed, the net effect is ambiguous because the switchgrass return has a lower reversion speed parameter but a higher difference between current return and long-term average.

The asymmetric pattern of the MR returns in figure 1.5a arises from the interaction of the three effects. For conversion boundary $b^{CS}$, when corn-soybean returns are low the distant time horizon effect dominates the other two effects, lowering the boundary. But when corn-soybean returns are high, the uncertainty effect dominates and the boundary is raised. For boundary $b^{CS}$, the three effects work together to lower the boundary when the corn-soybean returns are low, and the mean reversion effect dominates at high corn-soybean return levels.

Consistent with the conversion boundaries in figure 1.5a, figure 1.5b shows that the predicted proportion of land in switchgrass is higher under MR for the first 4 years, since the conversion boundary into switchgrass is low initially. As the returns grow over time, the proportion of land in switchgrass declines and becomes lower than under GBM processes. Eventually the switchgrass land proportion under both processes stabilizes around 16%.

*Effects of Conversion Costs*
Figure 1.6a shows how halving the cost of conversion costs affects the optimal conversion rule under the GBM assumption. In the top panel, reducing the conversion costs from corn-soybean to switchgrass ($C_{CS}$) creates the desired incentive by making the corn-soybean grower less reluctant to make the conversion. However, it also has the indirect effect of making the switchgrass grower more prone to convert (back) to corn-soybean, because although the farmer currently growing switchgrass will not directly benefit from the subsidy for conversion to switchgrass, its existence reduces the expected cost of converting from corn-soybean back to switchgrass, thereby reducing the implied cost of switching back to corn-soybean. Thus it indirectly increases his incentive to convert land to corn-soybean. The direct effect of lowering $C_{CS}$ is greater than the indirect effect. The reduction in $C_{CS}$ lowers the boundary from corn-soybean to switchgrass more than the boundary from switchgrass to corn-soybean. Similarly, the reduction in $C_{SC}$ lowers both of the conversion boundaries but lowers the boundary from switchgrass to corn-soybean more than the boundary from corn-soybean to switchgrass. These results also hold under the MR assumption.

An important insight from this two-way model is that a policy to subsidize conversion to dedicated energy crops, such as the USDA Biomass Crop Assistance Program, can have the joint effect of encouraging conversion both into and away from the biomass crop. Figure 1.6b illustrates how the two effects interact over time. Reducing the conversion cost from corn-soybean to switchgrass ($C_{CS}$) leads to higher proportions of land in switchgrass. But after 5 years, more switchgrass land is converted back to corn-soybean, due to the higher incentive to switch back to corn-soybean. As time approaching end of 30 years , the predicted portion switchgrass land is only 1% higher than baseline. Further, lowering conversion cost into corn-soybean, $C_{SC}$, in fact promotes conversion into switchgrass for the first seven years. Finally, in
the long run, lowering $C_{cs}$ and lowering $C_{sc}$ have almost the same effects on land in switchgrass.

**Effects of Uncertainties**

As discussed earlier, higher uncertainties in either corn-soybean or switchgrass returns will cause the farmer to be more reluctant to take any conversion action. Figure 1.7a shows that doubling the variance parameter of corn-soybean return $\sigma_c$ (or switchgrass return $\sigma_s$) significantly raises the conversion boundary from switchgrass (or corn-soybean) to corn-soybean (or switchgrass), $b^{sc}$ (or $b^{cs}$), and slightly raise the conversion boundary from corn-soybean to switchgrass $b^{cs}$ (or $b^{sc}$). As argued by Sarkar (2003), high uncertainties do not automatically translate into fewer conversions: although conversion is undertaken only with “more extreme” returns with the higher boundaries, higher uncertainty levels also mean that “extreme returns” occur more frequently. Figure 1.7b shows that doubling $\sigma_c$ and doubling $\sigma_s$ have strikingly different impacts: the proportion of land in switchgrass increases significantly as $\sigma_c$ doubles, but decreases to nearly zero (at 0.06) in the long run as $\sigma_s$ doubles.

**Effect of Positive Correlated Crop Returns**

Here we examine how the land conversion will change if the corn-soybean return and switchgrass return are positively correlated instead of negative correlated. Positive correlation implies the returns will more likely move to the same direction. The change of relative values will be smaller than when the correlation is negative. Thus the farmer will have less incentive to wait and option value of waiting is reduced. With perfect correlation, the current land use will remain preferred forever so that the option value of the alternative use will disappear. As shown in figure 1.8a, correlation between the two crop returns changing from -0.23 to 0.3 moves the conversion
boundaries towards each other and reduces the inaction zone. However, this change does not translate to much difference in the proportion of land in switchgrass as (figure 1.8b) as the incentives for converting into switchgrass and converting out both increase.

*Effect of Changing Growth Rate of Switchgrass Return under GBM Assumption*

The growth rate of the two return series are imposed the same. We tested the results sensitivity to this assumption by varying the growth rate of switchgrass return from 0.01 to 0.07. In general, increasing in the growth rate of switchgrass return will lower the conversion boundary from corn-soybean to switchgrass and raise the conversion boundary from switchgrass to corn-soybean while decreasing in it will have the opposite effect. Increasing $\alpha_s$ from 0.03 to 0.07 will make the proportional land in switchgrass peak at 0.44 (1.5 times baseline) and stabilize at 0.22 (1.38 times baseline). Decreasing $\alpha_s$ from 0.03 to 0.01 will make the proportional land in switchgrass peak at 0.24 (84% of baseline) and stabilize at 0.13 (81% of baseline). We report here two selective cases, $\alpha_s=0.07$ and $\alpha_s=0.01$ in figure 1.9a, 1.9b and 1.9c.

*Conclusions and Discussions*

This study develops a real options framework to analyze the farmer’s land use decision between traditional annual crops and perennial energy crops. The study innovates from existing models of optimal conversion under the assumption of irreversible decisions by introducing a model for two-way conversion. The possibility of costly reversibility in crop production is illustrated using an annual corn-soybean crop rotation and perennial switchgrass representative alternative crop systems. Consistent with real options theory, the option value of sticking to the current land use delays converting land into switchgrass as well as converting out of it. By comparison with the real options results, an NPV model predicts that an optimizing farmer would be much more prone to convert land to switchgrass. A one-way real option model characterizing the land conversion
decision as irreversible predicts much greater reluctance to convert land from annual corn-
soybean to a perennial switchgrass energy crop, also implying lower accumulated land under
energy crops over a 30-year time horizon. We further show how two alternative stochastic
process assumptions affect the optimal conversion rule and the proportion of land devoted to the
dedicated energy crop.

From a policy perspective, this model offers two important insights. First, compared to
deterministic break-even analyses (e.g., Tyner 2008; James, Swinton, and Thelen 2010), it
highlights the significant option value of delaying land conversion even when a static net present
value threshold is passed. The illustrative case here suggests that returns from dedicated energy
crops may have to exceed double the breakeven NPV level before becoming a dynamically
optimized choice.

Second, compared with past real options models that assume complete irreversibility of
decisions, this two-way model reveals that conversion subsidies to encourage biofuel crop
planting have a two-edged impact. The effect of reducing conversion costs from corn-soybean to
an energy crop (switchgrass) is to lower the conversion threshold revenue levels in both
directions, meaning that not only is it easier to convert from corn-soybean into switchgrass, but it
also is easier to convert the other way. Compared to the case of no subsidy, the predicted
proportion of land planted to the switchgrass energy crop is higher with the subsidy in the
intermediate period but difference becomes negligible toward the latter part of a 30-year time
horizon.

One limitation of this model is that we did not incorporate the switchgrass age into land
conversion model. Several issues relate to the age of switchgrass (or more generally, an energy
crop) in determining farmer’s decision. The first is that farmer may face liquidity constraint
problem in growing energy crops since costs for establishment and reseeding incur in the first
several years after planting while the yields are lower at the same time. The net returns in the
beginning of the production cycle may be even negative. Second, although we use ten years as a production cycle in our paper because it is recommended by the agronomists and is used in most literature to calculate the switchgrass production costs, ten years may not be the optimal replanting time. The energy crop grower can choose the optimal replanting time subject to the uncertainties and has the option to convert land to an alternative use. Third, while the conversion costs from switchgrass to other crop stay fairly constant over years since its roots do not change much, this is not true for some other energy crops, such as miscanthus. As the roots grow over time, the conversion costs from miscanthus to other crops grow over time too. Introducing the age of energy crops into the model can help (at least partially) address the above issues and will be an extension of our model.
Table 1.1 Baseline Parameters for Numerically Solving the Dynamic Optimal Land Conversion Rule

**Parameters of the stochastic processes of the returns to corn-soy and switchgrass**

<table>
<thead>
<tr>
<th></th>
<th>Returns to corn-soy</th>
<th>Returns to switchgrass</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GBM</strong></td>
<td></td>
<td></td>
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<tr>
<td>Drift parameter</td>
<td>$\hat{\alpha}_c$</td>
<td>$\hat{\alpha}_s$</td>
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</tr>
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<td>$\hat{\pi}_s$</td>
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<td>-0.23</td>
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</tbody>
</table>

**Land conversion costs**

- Corn-soy to switchgrass $C_{CS}$: 136 $/acre
- Switchgrass to corn-soy $C_{SC}$: 47$/acre

**Discount factor** $r$ 0.08
Figure 1.1 Conversion boundaries
Figure 1.2 Average returns to corn-soybean and switchgrass in North-central U.S. (1982 - 2008, in 1982 dollars)
Figure 1.3a NPV vs. Dynamic optimal (under GBM): conversion boundaries
Figure 1.3b  NPV vs. Dynamic optimal rule (under GBM): proportion of land in switchgrass
Figure 1.4a Two-way vs. one-way conversion (under GBM): conversion boundaries
Figure 1.4b Two-way vs. one-way conversion (under GBM): proportion of land in switchgrass
Figure 1.5a GBM vs. MR: conversion boundaries
Figure 1.5b  GBM vs. MR: proportion of land in switchgrass
Figure 1.6a Reducing conversion costs (under GBM): conversion boundary
Figure 1.6b Reducing conversion costs (under GBM): proportion of land in switchgrass
Figure 1.7a Return volatility (under GBM): conversion boundary
Figure 1.7b  Return volatility (under GBM): proportion of land in switchgrass
Figure 1.8a Return correlation (under GBM): conversion boundary
Figure 1.8b. Return correlation: proportion of land in switchgrass
Figure 1.9a Increasing growth rate to 0.07 (under GBM): conversion boundary
Figure 1.9b Decreasing growth rate to 0.01 (under GBM): conversion boundary
Figure 1.9c Varying growth rate (under GBM): proportion of land in switchgrass
Appendix A
Homogenous Two-way Land Conversion Model

Model Solution

In this appendix, we solve a two-way land conversion model, of which the value functions are homogenous of degree one in the two return variables. The key assumption to make the value functions homogenous is that conversion costs are proportional to the return to current use, $C_{ij} = k_{ij}\pi_i$. The other assumptions are maintained the same as in table 1.1.

At time $t$, if the farmer chooses between keeping the land in use $i$, the value function has to satisfy the Bellman equation A(1). Left hand side (LHS) is the market rate of return of investing $V^i$ dollars while right hand side (RHS) is the rate of return generated from the land use $i$, which equals the instantaneous return $\pi_i(t)$ plus the change of value function. The no arbitrage condition requires the RHS equals LHS.

$$\text{A}(1) \quad rV^i(\pi_c, \pi_s) = \pi_i(t) + \frac{EdV^i(\pi_c, \pi_s)}{dt}$$

By Ito’s Lemma, it can also be written as a partial differential equation

$$\text{A}(2) \quad rV^i(\pi_c, \pi_s) = \pi_i(t) + \alpha_c\pi_c V^i + \alpha_s\pi_s V^i + 1/2\sigma_c^2\pi_c V^i + 1/2\sigma_s^2\pi_s V^i + \rho\sigma_c\pi_c\sigma_s\pi_s V^i$$

where the subscripts denote the partial derivatives. Equation A(2) captures the relationship between the two state variables $\pi_c$ and $\pi_s$. It must be satisfied by $V^i(\pi_c, \pi_s)$ if the land is currently in use $i$. We can also observe that value functions should be homogenous of degree one in $\pi_c$ and $\pi_s$. Thus the optimal land conversion rule depends only on the ratio of the two land
returns. We define \( d_{ij} = \frac{\pi_j}{\pi_i} \), then the value functions can be written as \( V^i = \pi_i \cdot g^i(d_{ij}) \).

Successive differentiation gives

\[
V^i_{\pi_i} = g^i(d_{ij}) - d_{ij}g^i_d(d_{ij}) \\
V^i_{\pi_j} = g^i_{d_{ij}}(d_{ij})
\]

(A3) \[
V^i_{\pi_i\pi_i} = \frac{d^2_{ij}}{\pi_i} g^i_{d^2_{ij}}(d_{ij}) \\
V^i_{\pi_j\pi_j} = -g^i_{d_{ij}d_{ij}}(d_{ij})(\pi_j/\pi_i^2)
\]

Substituting these derivatives into equation A(2) and grouping terms, we obtain two ordinary differential equations for each unknown function \( g^i(d_{ij}) \).

(A4)

\[
1/2 \sum_{i=c,s} \sigma_i^2 d^2_{ij} g^i_{d^2_{ij}}(d_{ij}) + (u_j - u_i)g^i_{d_{ij}}(d_{ij}) + (u_i - r)g^i(d_{ij}) + 1 = 0
\]

Since each of the second order differential homogenous equations in (A4) is linear in the dependent variables \( d_{ij} \) and its derivatives, its solution can be expressed as a sum of a general solution and a particular solution:

(A5)

\[
g^i(d_{ij}) = A_{i1}d^\beta_{i1} + A_{i2}d^\beta_{i2} + 1/(r - \alpha_i)
\]

Where \( A_{i1} \) and \( A_{i2} \) are constants to be determined and \( \beta_{i1} \) and \( \beta_{i2} \) are the positive and negative roots of the following fundamental quadratic equation
$1/2(\sigma_c^2 + \sigma_s^2 - \rho \sigma_c \sigma_s)\beta_l(\beta_l - 1) + (\alpha_j - \alpha_i)\beta_i + (\alpha_i - r) = 0$

Correspondingly the value function $V^i$ can be expressed as

$$V^i(\pi_c, \pi_s) = \pi_i(A_{i1}d_{ij}^{\beta_i1} + A_{i2}d_{ij}^{\beta_i2}) + \pi_i/(r - \alpha_i)$$

There is a nice economic interpretation about $A(7)$: the second term is the expected NPV if the status quo is maintained forever while the first term is the value to have the option to convert land to use $j$.

Up to now we have characterized the conditions that the value functions need to satisfy to stay in the current use. To fully solve the optimal land conversion rule, we need to find the constants $A_{c1}$, $A_{c2}$, $A_{s1}$, and $A_{s2}$ as well as the critical land conversion ratio $d_{cs}^*$ and $d_{sc}^*$. This requires the value functions to satisfy three additional boundary conditions.

$$V^i(0, \pi_j) = 0 \quad (6)$$

$$V^i(\pi_c, \pi_s) = V^j(\pi_c, \pi_s) - k_{ij}\pi_i \quad \text{when} \quad \pi_j = \pi_j^*(\pi_i) \quad (9)$$

$$V^i(\pi_c, \pi_s) = V^j(\pi_c, \pi_s) - k_{ij} \quad \text{when} \quad \pi_j = \pi_j^*(\pi_i) \quad (10)$$

The first boundary condition $A(8)$ is derived from the fact that $V^i(\bullet)$ will be zero if the $\pi_i$ goes to zero since $V^i = \pi_i^* g^i(d_{ij})$. This implies $A_{i2} = 0$. The other two conditions come from consideration of optional conversion (see details in Dixit and Pindyck (1994)), chapter 4). When
\( \pi_j > \pi_i \), converting land from use \( i \) to \( j \) is optimal if \( \pi_j \) is greater than the critical boundary \( \pi_j(\pi_i) \) (denoted by \( b_{ij}^* \)). As we noted above, the decision depends only on the ratio

\[
d_{ij}^* = \frac{\pi_j(\pi_i)}{\pi_i},
\]

which is the slope of the conversion boundary. On the conversion boundary, the value functions need to satisfy the value matching conditions and smooth pasting condition.

Value matching conditions says that on conversion boundary the farmer is indifferent between staying in current land use and converting to alternative use. Smooth pasting condition impose the equality of marginal changes in the continuation value and the stopping value when

\[
\pi_j = \pi_j(\pi_i).
\]

Now substituting the \( A(5) \) into equation \( A(9) \) and \( A(10) \) for \( i = c, s \) respectively and grouping terms, we can obtain a system of four equations with four unknowns, \( A_{c1}, A_{s1}, d_{cs}^* \) and \( d_{sc}^* \).

Since the term \( A_{it}^2d_{ij}^i \) is eliminated, we depress the notation 1 in the \( A_{it} \) and \( \beta_{it} \) and simply use \( A_i \) and \( \beta_i \) in the following.

\[
A_c(d_{cs}^*)\beta_c - A_s d_{cs}^*(1-\beta_s) + 1/(r - \alpha_c) - d_{cs} l(r - \alpha_s) + k_{cs} = 0
\]

\[
\beta_c A_c(d_{cs}^*)(\beta_c - 1) - (1 - \beta_s) A_s d_{cs}^* - \beta_s - d_{cs} l(r - \alpha_s) = 0
\]

\[
A_c(d_{sc}^*)(1 - \beta_c) - A_s d_{sc}^* \beta_s + d_{sc} l(r - \alpha_c) - 1/(r - \alpha_s) + k_{sc} = 0
\]

\[
(1 - \beta_c) A_c(d_{sc}^*) - \beta_c A_s d_{sc}^* (\beta_s - 1) + 1/(r - \alpha_c) = 0
\]
The critical ratio of land conversion can be solved after parameterize the system of equations $A(11)$. We use the same parameters as in Essay 1 table 1.1 except that the conversion costs parameters. The $k_{CS}$ and $k_{SC}$ are assumed to be 0.9 and 0.3 respectively. Given these parameters, the critical return ratio of $d_{CS}^*$ to induce farmer to convert land from corn-soy to switchgrass is 2.39. The critical return ratio of $d_{SC}^*$ for farmer to make the opposite conversion is 2.12. As expected, the critical ratios are higher than 1 due to uncertainty and costs. The farmer with an opportunity to convert land to an alternative use (switchgrass) is like holding an “option”, which enables him to wait for more information to arrive and make more informed decision. Once he converts, it is costly to convert back to original land use (corn-soybean) if the market conditions turn out to be less favorable. Thus, sticking to current land se has an additional value, called “option value”, derived from converting to alternative land use (switchgrass). Once the farmer converts land to switchgrass, there is a similar option value associated with converting back to corn-soybean. These two option values are mutually dependent and need to be solved simultaneously as shown in the conditions $A(9)$ and $A(10)$.

The result is shown in figure A1 where the switchgrass return increases from zero to $500/acre. Each critical boundary separates a waiting region (staying in current use) from an exercise region (converting to alternative use). As shown in figure A1, the two conversion boundaries starting from origin divide the $\pi_C - \pi_S$ region into three parts: above $b^{CS}$, it is optimal to convert land from corn-soybean to switchgrass; below $b^{SC}$, it is optimal to convert land from switchgrass to corn-soybean; between them, it is optimal to stay in current use. Once the land is already in one use, the farmer is reluctant to convert to the alternative use unless the return from the latter is sufficiently high. There is a large inaction zone between the conversion boundaries.
**Comparative Statics**

We can examine the sensitivity of the critical return ratios $d_{CS}^{*}$ and $d_{SC}^{*}$ to the change of the key parameters. In essay 2 we will analyze the effects of the subsidies, which could change the dynamics of the switchgrass return. These comparative statics can help explain the later simulation results.

Different from a one-way land conversion model, any change associated with one land use return in a two-way land conversion model, such as the variance or conversion costs, will affect the land use decision rules in both directions. First we vary variance $\sigma_s$ from 0.1 to 1, holding other parameters unchanged. A rise in the uncertainty over switchgrass return will increase the option value, not only to convert land into the switchgrass but also to convert out. Therefore it requires higher current thresholds of return ratios, $d_{CS}^{*}$ and $d_{SC}^{*}$, to make both direction conversions. The results are shown in figure A2.

Second, we vary drift rate of switchgrass return, $\alpha_s$, from 0.01 to 0.07. The drift rate in the GBM represents how fast the stochastic variable is expected to grow. Anticipating a faster growth rate of switchgrass return the farmer growing corn-soy is more willing to convert land to switchgrass while farmer growing switchgrass is less willing to withdraw land from it. Therefore, critical ratio of converting into switchgrass, $d_{CS}^{*}$, is raised while the critical ratio of converting out of switchgrass, $d_{SC}^{*}$, is lowered (figure A3).

Third, we examine the effect of conversion costs on optimal conversion rules by varying the proportional parameter of conversion costs from corn-soybean to switchgrass, $k_{CS}$, from 0.1 to 1. Clearly, the higher the conversion cost from corn-soybean to switchgrass is, the more costly to
falsely make the conversion. The corn-soybean grower will therefore be more reluctant to convert to switchgrass. Moreover, higher $k_{CS}$ increases the expected cost of converting from corn-soybean back to switchgrass, thereby increasing the implied cost of switching to corn-soybean. Thus it indirectly depresses the switchgrass grower’s incentive to convert land to corn-soybean.

The direct effect of increasing $k_{CS}$ on critical ratio of converting into switchgrass is larger than the effect of the indirect effect on critical ratio of converting back to corn-soybean. As shown in figure A4, $d_{CS}^*$ rises faster than $d_{SC}^*$.

Last, we examine the effect of return correlation. In the absence of uncertainty, the coefficient of correlation is meaningless since land returns do not change. Under uncertainty, a positive correlation implies the returns will more likely move to the same direction. The change of relative values will be smaller than when the correlation is negative. Thus the farmer will have less incentive to wait and option value of waiting is reduced. With perfect correlation, the current land use will remain preferred forever so that the option value of the alternative use will disappear. As shown in figure A5, correlation between the two crop returns changing from -0.5 to 0.5 reduces the critical return ratio of both directions. This moves the conversion boundaries towards each other and reduces the inaction zone.
Figure A.1 The critical land conversion boundaries
Figure A.2 Sensitivity of the critical return ratios to the variance of switchgrass return
Figure A.3 Sensitivity of the critical return ratios to the drift rate of switchgrass return
Figure A.4 Sensitivity of the critical return ratios to conversion costs
Figure A.5 Sensitivity of the critical return ratios to correlation coefficient between two return series
Appendix B
Numerical Solution for the Optimal Land Use Decision Model

The numerical solution is implemented in Matlab using OSSOLVE solver (Fackler 2004) and CompEcon toolbox (Miranda and Fackler, 2002). More reference can be found in Brekke and Brent Oksendal (1994) and Qi and Liao (1999). Our multi-states and dimensions optimal land conversion problem (3) can be reduced to a set of optimal conditions that value functions satisfy Bellman differential equations and boundary conditions respective to different states:

\begin{align}
\mathbf{B}(1)
& rV^i(\pi_c, \pi_s) - \pi_c(t) - \alpha_c(\pi_c, t)V^i_{\pi_c} - \alpha_s(\pi_s, t)V^i_{\pi_s} - 1/2\sigma_c^2(\pi_c, t)V^i_{\pi_c^2} \\
& - 1/2\sigma_s^2(\pi_s, t)V^i_{\pi_s^2} - \rho\sigma_c(\pi_c, t)\sigma_s(\pi_s, t)V^i_{\pi_c\pi_s} \geq 0 \\
& i = c, s
\end{align}

\begin{align}
\mathbf{B}(2)
& V^i(\pi_c, \pi_s) \geq V^j(\pi_c, \pi_s) - C_{ij} \\
& i, j \in S \text{ and } i \neq j
\end{align}

(iii) either of the (i) or (ii) has to hold as strict equality.

We cannot obtain the explicit form of value functions and have to rely on the numerical solution methods. This problem falls into a more general category of so called functional equation problem, e.g., we need to find a function \( f \) that satisfies:

\begin{align}
\mathbf{B}(2)
& Tf = 0
\end{align}

where \( T \) is an operator that maps a vector space of function into itself. The collocation method is a function approximation method widely used to solve functional equations and will be employed here (Judd, 1998; Miranda and Fackler, 2002).

\textit{Collocation method} is to approximate unknown function \( f \) by using a linear combination of \( n \) pre-defined basis functions drawn from a family of approximating functions:
\[ \hat{f}(x) = \sum_{j=1}^{n} c_j \phi_j(x) \]. The n coefficients \( c_1, c_2, \ldots, c_n \) can be found by solving a system of n equations resulting from evaluating the approximants at n nodal points \( x_1, x_2, \ldots, x_n \) and requiring them to satisfy the functional equations:

\[ B(3) \quad g(x_i, \sum_{j=1}^{n} c_j \phi_j(x)) = 0 \quad \forall i = 1, 2, \ldots, n \]

The collocation method thus can approximate the function indirectly by using the rootfinding techniques for non-linear equations, for example, Newton’s method or quasi-Newton methods.

Specifically, the value function \( V^i(\pi_c, \pi_s) \) can be approximated by a linear combination of \( n (n_c \times n_s) \) known basis function.

\[ B(4) \quad \hat{V}^i(\pi_c, \pi_s) = \sum_{j_c=1}^{n_c} \sum_{j_s=1}^{n_s} c_{j_c} j_s \phi_{j_c} j_s (\pi_c, \pi_s) \]

where \( \phi \) represents a set of n basis functions for a family of approximation functions and \( \theta^i \) is a vector of coefficients for the value functions associated with the \( i \)th state. The family basis function we use is piecewise linear spline with a 101-degree on the interval \( \{(\pi_c, \pi_s) | 0 \leq \pi_c \leq 10, 0 \leq \pi_s \leq 10\} \). More details about how to define basis function can be found in Miranda and Fackler (2002, page 136-149).

Define the approximation differential operator:
\[ \beta^i(\pi_c, \pi_s) = r\phi(\pi_c, \pi_s) - \alpha_i(\pi_i, t)\phi_{\pi_i}^i(\pi_c, \pi_s) \]

B(5)

\[ -1/2\sigma_i^2\phi''_{\pi_i\pi_i}(\pi_i, t) - \rho\sigma_c\sigma_s\phi''_{\pi_i\pi_i}(\pi_i, t) \]

Now the optimality conditions B(1) can be written as

(i) \( \beta^i(\pi_c, \pi_s)\theta^i - \pi_i \geq 0 \quad i = 1, 2 \)

B(6)

(ii) \( \phi(\pi_c, \pi_s)\theta_i - \phi(\pi_c, \pi_s)\theta_j - C_{ij} \geq 0 \quad \forall i \neq j \)

(iii) either of the (i) or (ii) has to hold as strict equality.

Define the matrix \( \Phi \) and \( B^i \) as the function \( \phi(\pi_c, \pi_s) \) and \( \beta^i(\pi_c, \pi_s) \) evaluated at the state variables nodal values. The optimality conditions (1) can be reformulated as an Extended Vertical Linear Complementarity Problem:

B(7)

\[ G(z) = \min(M_c z + q_c, M_s z + q_s) = 0 \]

where \( z \) is the state associated coefficients \( \theta \) stacked vertically. The \( M \) and \( q \) is the given by

B(8)

\[ M_c = \begin{bmatrix} B^c & 0 \\ -\Phi & \Phi \end{bmatrix} \quad q_c = \begin{bmatrix} -\pi_c \\ -\pi_c \end{bmatrix} \quad M_s = \begin{bmatrix} \Phi & -\Phi \\ 0 & B^s \end{bmatrix} \quad q_s = \begin{bmatrix} C_{cs} 1_n \\ -\pi_s \end{bmatrix} \]

where \( 1_n \) is a column vector composed of \( n \) ones and the min operator is applied element wise.

The collocation method can be used to solve for \( z \). Notice that the function \( G(z) \) in EVLCP problem is not continuously differentiable, thus standard Newton method is not applicable. Qi and Liao (1999) proposed a smoothing Newton method by using the aggregation technique.

Rewrite \( G(z) \) as \( -\max\{-(M_c z + q_c), -(M_s z + q_s)\} \) and define an aggregation function \( G(t, Z) \):
The smoothing parameter \( t \) is defined as the first component of the following equation:

\[
(G(t, Z))_t = \begin{cases} 
-|t| \ln \left( \sum_{j=c,s} \exp(-M_{j}z + q_{j}) / |t| \right) & \text{if } t > 0 \\
G(z) & \text{if } t = 0 
\end{cases}
\]

The equation \( H(t, Z) \) can be solved by invoking Newton method and enforcing the parameter \( t \) positively converging to zero. The point is to approximate the nonsmooth function \( G(z) \) by a strongly semismooth function \( H(t, Z) \) by using the aggregation technique. The solution of the \( G(t, Z) \) is the same as the function \( H(t, Z) \) when \( t \) is equal to 0. The function \( H(t, Z) \) can be approximated by the Newton method for every parameter \( t > 0 \) and gradually reduce \( t \) to zero. Qi and Liao proved that when \( t \) approaches zero, the solution to \( H(t, Z) \) approaches to the solution to \( G(z) = 0 \). Based on the algorithm proposed by Qi and Liao, Falker (2004) implement the algorithm in Matlab coded a procedure OSSOLVE.
Appendix C  
Specification Tests

Testing Unit Root: Dicky-Fuller Test and Phillips-Perron Test

Dicky-Fuller (DF) test requires to run the regression

\[ \Delta \ln \pi_{i,t} = \beta_0 + \beta_1 \ln \pi_{i,t} + \epsilon_{i,t}, \quad i = c, s. \]

The null hypothesis is the \( \ln \pi_{i,t} \) has a unit root or \( \beta_1 = 0 \). There is no evidence of AR(1) serial correlation in the logarithms of both crop returns. To address other type of potential serial correlations, we perform Philips-Perron (PP) test (Phillips and Perrorn 1988), which requires to run the regression on

\[ \ln \pi_{i,t} = \gamma_0 + \gamma_1 \ln \pi_{i,t-1} + u_{i,t}, \quad i = c, s. \]

The null is \( \ln \pi_{i,t} \) has a unit root or \( \gamma_1 = 1 \). For logarithm of return to corn-soybean, both DF test and PP test can not reject the unit root. Similar results are found for the logarithm of return to switchgrass.

Testing Stationarity: KPSS Test

The KPSS test (Kwiatkowski et al. 1992) can have a null hypothesis of either level stationary or trend stationary. We choose the level stationary as our null hypothesis. For both logarithms of returns to corn-soybean and switchgrass, the null can not be rejected at 10% level.

Testing Serial Correlations and Heteroskedasticity

To estimate the parameters in GBM and MR processes, we discretize them into equations (9) and (11), in which conditional variance term is assumed to be serial uncorrelated and constant. Here we test whether the error terms show AR(1) type serial correlation and ARCH type of heteroskedasticity. Following Wooldridge (2000), the steps of testing for AR(1) serial correlation in GBM are as following:

1) Run the regression (9) save the residues \( \hat{\epsilon}_{i,t} \) for all \( t \).
2) Run the regression of \( \hat{\epsilon}_{i,t} \) on \( \hat{\epsilon}_{i,t-1} \) and explanatory variables in (9).

3) If the coefficient of \( \hat{\epsilon}_{i,t-1} \) is statistically different from zero, then there is sign of serial correlation.

The steps of testing for heteroskedasticity are as following:

1) Run the regression (9) save the residues \( \hat{\epsilon}_{i,t} \) for all \( t \).

2) Run the regression of \( \hat{\epsilon}_{i,t}^2 \) on \( \hat{\epsilon}_{i,t-1}^2 \) (including constant).

3) If the coefficient of \( \hat{\epsilon}_{i,t-1}^2 \) is statistically different from zero, then there is sign of ARCH(1).

The same steps apply to MR process (equation 11). Table A4 and A5 show the testing results. In summary, there is no evidence of serial correlations and ARCH(1) for the two returns under both GBM and MR assumptions.
### Table C.1 Simple Dicky-Fuller Test Results

<table>
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<tr>
<th>Lagged logarithm of return to corn-soybean</th>
<th>Coefficient $\beta_1$ and t-statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged logarithm of return to switchgrass</td>
<td>-0.31 (-2.16)</td>
</tr>
</tbody>
</table>

Dickey-Fuller critical values: 1%, -3.45; 5%, -2.87; 10%, -2.57.
Table C.2 Phillips-Perron Test Results

<table>
<thead>
<tr>
<th></th>
<th>Test statistics for $\gamma_1 = 1$</th>
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<tbody>
<tr>
<td>Lagged logarithm of return to corn-soybean</td>
<td>-2.07</td>
</tr>
<tr>
<td>Lagged logarithm of return to switchgrass</td>
<td>-2.39</td>
</tr>
</tbody>
</table>

Critical values: 1%, -3.743; 5%, -2.997; 10%, -2.629.
### Table C.3 KPSS Test Results

<table>
<thead>
<tr>
<th>Test Statistics on 8th Lag</th>
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<tbody>
<tr>
<td>Logarithm of return to corn-soybean</td>
<td>0.34</td>
</tr>
<tr>
<td>Logarithm of return to switchgrass</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Maxlag = 8 chosen by Schwert criterion. Critical values for $H_0$: $\log \pi_{i,t}$ is trend stationary: 1%: 0.74; 5%: 0.46; 10%: 0.35.
<table>
<thead>
<tr>
<th></th>
<th>Coefficients and t-statistics on Lagged Errors</th>
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<tr>
<td></td>
<td>GBM</td>
<td>MR</td>
<td></td>
</tr>
<tr>
<td>Corn-soybean</td>
<td>-0.37 (-1.36)</td>
<td>-0.38 (-1.67)</td>
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</tr>
<tr>
<td>Switchgrass</td>
<td>-0.20 (-0.64)</td>
<td>-0.22 (-1.02)</td>
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Table C.5 Heteroskedasticity ARCH(1) Test

<table>
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<tr>
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<tr>
<td>Corn-soybean</td>
<td>0.36 (1.91)</td>
</tr>
<tr>
<td>Switchgrass</td>
<td>0.19 (0.94)</td>
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References


Essay 2
Alternative Land Use Policies: Real Options with Costly Reversibility

Introduction

Agricultural subsidies have been used to induce socially desirable land uses for a long time. An example is the U.S. Conservation Reserve Program, in which farmers set aside production land to provide environmental benefits and receive payments from government in return. One strategy to mitigate climate change proposed in the United States and Canada is subsidizing farmers to convert the marginal agricultural land to forest for more carbon sequestration (Stavins 1999; McKenney et al. 2004; Lubowski, Plantinga, and Stavins 2006). A body of literature has analyzed the effects of subsidies on the land use change, such as Stavins and Jaffe (1990) and Plantinga, Mauldin and Miller (1999). A common assumption in these studies is that a farmer will compare the expected net present value (ENPV) of returns to different land uses and choose the one with the highest ENPV. Thus subsidizing a desirable land use will raise its return and induce land converted to it. ENPV decision rule implies that the form of the subsidy, such as lump sum or continuous, constant or variable, does not matter. Subsidies that are equal under the ENPV rule are implicitly assumed to give farmers the same incentive to convert land to the desirable use.

It has been observed that farmers often do not convert land even if it is profitable to do so under the ENPV rule (Isik and Yang 2004; Plantinga et al. 2002). Parks (1995) explained land conversion hysteresis as a consequence of risk aversion and expected capital gains. He also explored the effects of some types of conversion subsidies. Although not explicitly clear, in his model a cost-sharing subsidy and a constant annual rental payment, which do not change the uncertainties of land returns, can give a farmer the same conversion incentive if their annualized values are equal. In contrast, the real options framework shows that the interaction among irreversible sunk cost, uncertainty, and learning can cause even risk-neutral farmers to be more reluctant to change land uses than the ENPV rule predicts (e.g., Titman 1985). More importantly,
subsidies taking various forms can affect the conversion costs, the level and uncertainty of land use returns differently. Subsequently they will affect the farmers’ land conversion decision differently even though they are ENPV-equivalent.

The purpose of this paper is to compare the long term effectiveness of different forms of agricultural subsidies in achieving an increase in a desired land use. We adopt an innovative real options approach by relaxing the absolute irreversibility assumption in previous literature and allowing for land use conversion in two directions. A farmer deciding on converting to another land use is allowed to take into consideration the future possibility of converting the land back to its original use under plausible market conditions. The absolute irreversibility assumption might be reasonable for urban development (Capozza and Li 1994; Abebayehu, Keith, and Betsey 1999). However for agricultural land, a farmer can switch between different uses with costs. Allowing two-way land conversion can help capture the flexibility of farmer’s land use decisions. Moreover, it has important implications in designing subsidy programs since the subsidies not only change the farmer’s willingness to convert land into the desirable use but also the willingness to convert it out.

To make our ideas concrete, we evaluate land conversion subsidies in the context of encouraging production of energy crops, which can be directly combusted to provide electricity or converted to transportation fuel. Globally the market demand for energy crops is largely driven by various renewable energy policies. For example, in the United States, more than 20 states mandate Renewable Portfolio Standards (RPS), which require a certain minimum quantity of electricity produced from eligible renewable energy sources. Biomass is an eligible energy source in some states. But more (potential) demand for energy crops may come from cellulosic biofuel production. Currently liquid biofuels are strongly advocated in many countries, including the United States, due to political concerns related to energy security, climate change and rural development (Khanna 2008; Rajagopal and Zilberman 2007). Although grain-based biofuel currently dominates the market, cellulosic biofuel is believed to have superior environmental
performance, such as higher net energy, higher carbon credit and more-environmental friendly-feedstocks (Schmer et al. 2008; Paine et al. 1996). For this reason, the U.S. Energy Independence and Security Act (EISA) of 2007 mandates the use of cellulosic ethanol, increasing from 0.1 billion gallons annually in 2010 to 16 billion gallons in 2022. To meet this mandate, significant expansion of energy crops is expected to occur on agricultural land and compete with traditional crops for the limited acres (Thomson et al. 2008; Walsh et al. 2003).

Coupled with energy policies that induce energy crop production through creating new markets for them, many countries also use agricultural subsidies to provide direct production incentives. The perennial nature of most energy crops involves sunk costs to establish the plants, which may become prohibitively high for some woody crops. To overcome this barrier, a lump-sum payment is often provided to cover the establishment costs in full or partial. In the 1990s, Sweden offered 10,000SEK/ha (roughly $573/acre) establishment subsidy for planting willow (Helby, Rosenqvist, and Roos 2006). In early 2007, the Irish government announced it would subsidize half of the establishment costs for willow and miscanthus (Styles, Thorne, and Jones 2008). In the United States, the Food, Conservation and Energy Act (FCEA) of 2008 introduced direct payments for up to 75% of establishment costs for eligible energy crops. In addition to a cost-sharing subsidy to help start-up, annual payment is also provided by governments to support production, collection, harvest, storage and transportation of energy crops. For example, European Union (EU) farmers can receive an annual payment of €45/ha (roughly $25/acre) for growing energy crops on production land (Rajagopal and Ziberman 2007). The Irish government subsidizes additional €85/ha (about $45/acre) for growing willow and miscanthus (Styles, Thorne, and Jones 2008). In contrast, the U.S. farmers can receive a payment to cover costs of harvest, storage and transportation that is equal to what they obtain from biorefiners for 2 years (up to $45/ton). This type of subsidy will vary with the market price and yield of biomass. FCEA also required the Federal Crop Insurance Cooperation to study the insurance policies for energy crops, providing the future possibility of subsidizing the energy crops insurance. Given that large
subsidy amounts are spent and take different forms, the effectiveness of these subsidies should be systematically evaluated.

Our paper contributes to the literature in several aspects. First, we examine a range of ENPV-equivalent subsidies, showing how they can affect a representative farmer’s optimal land conversion rule differently, depending on their effects on the conversion costs, returns level and variability of returns. Second, we examine how optimal land conversion strategies differ between a real options framework assuming irreversible land use decisions and our framework, which allows reversion to a prior land use. In this framework, it turns out that subsidies not only change the farmer’s willingness to convert land into energy crops but also the willingness to convert back out. Third, based on this improved model, we compare the effectiveness of subsidy programs for encouraging the production of energy crops.

The remainder of the paper is organized as follows. In the next section, we present a general land conversion decision model without governmental intervention to better expose the idea of uncertainty and sunk costs causing hysteresis in land conversion. Next, we examine how various forms of subsidies for energy crops can change a representative farmer’s land conversion decision rule differently even though they are ENPV-equivalent. Then we perform a Monte Carlo simulation on the farmer’s annual land use choice under each type of subsidy over a period of 30 years. The subsidy levels are calibrated so that they have the same expected cost to the governmental and their long-term performances are compared according to the increased expected conversion rate into energy crops than no subsidy support. Finally we give results and conclusions.

**Land Conversion Decision Model**

*Decision without Governmental Intervention*
Consider a representative, risk neutral farmer with a unit of land facing two competing crop production alternatives: a corn-soybean rotation and switchgrass, which are selected as representative of a traditional crop and an energy crop. The returns to corn-soybean and switchgrass at period $t$ are denoted by $\pi_c(t)$ and $\pi_s(t)$, respectively. The farmer can convert land from corn-soybean to switchgrass with a lump-sum cost $C_{cs}$ or vice versa with a lump-sum cost $C_{sc}$. The farmer seeks to maximize the net present value of current and future returns at a discount rate $r$ over an infinite time horizon. The future returns to corn-soybean and switchgrass are assumed to evolve according to geometric Brownian motion (GBM) $^{15}$:

$$d\pi_i = \alpha_i \pi_i dt + \sigma_i \pi_i dz_i$$

where $dz_i$ is the increment of a Wiener process. The correlation coefficient of the two return processes is $\rho$, i.e., $E(d\pi_c dz_s) = \rho dt$. Traditional crop and energy crop returns could be correlated for a variety of reasons, e.g. both are linked with energy prices and are subject to macro-economic shocks. $^{16}$

According to the ENPV decision rule, the farmer will switch from one crop to another when the ENPV of switching is higher than staying, i.e.,

$$\int_0^\infty \pi_j(t)e^{-rt} dt - C_{ij} \geq \int_0^\infty \pi_i(t)e^{-rt} dt \quad i, j \in \{c, s\}, \text{ and } i \neq j$$

The real options literature has pointed out that the ENPV approach ignores that the agent can optimally postpone their irreversible actions. In this case, the farmer with an opportunity to switch crop is like holding an “option”. When he exercises this option (convert to another crop), he gives up the opportunity of waiting for new information that enables him to make more informed decision. It is costly to revert it back to original crop if the market conditions turn out to

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$^{15}$ We drop off the time $t$ to simplify the notation whenever it does not cause confusion.

$^{16}$ Land switching between the two crops could make the crop prices endogenous. However, explicitly modeling this relationship is beyond the scope of our paper.
be undesirable. The lost option value is an opportunity costs that must be include as part of cost of land conversion.

Next we derive the optimal decision using real options approach. Let $V^i(\pi_c, \pi_s)$ be the value function of currently being in land use $i$, which is defined as the expected net present value of all future returns starting from corn-soybean and then following optimal policies. Due to the option of converting into use $j \neq i$, the payoff depends on the distribution of future returns of both land uses, the information for which is contained in the two current returns, $\pi_c(t)$ and $\pi_s(t)$. At time $t$, the farmer chooses between keeping the land in use $i$ and converting it into alternative use $j$:

$$\begin{align*}
(3) \quad V^i(\pi_c, \pi_s) &= \max \left\{ \pi_i(t) dt + e^{-rdt} EV^i(\pi_c(t + dt), \pi_s(t + dt)),
\right. \\
&\left. \quad V^j(\pi_c(t), \pi_s(t)) - C_{ij} \right\}
\end{align*}$$

The first term on the right hand side describes the payoffs if the land is kept in use $i$: in the infinitesimal period $[t, t + dt]$, the farmer receives profit from land use $i$ at rate $\pi_i(t)$, and at the end of the period, receives the new discounted expected payoff $e^{-rdt} EV^i(t + dt)$. The second term on the right hand side describes the payoff if the land is converted into use $j$: the farmer receives the expected payoff of use $j$, $V^j(t)$, but incurs the conversion cost $C_{ij}$.

Intuitively, the conversion decision will depend on the relative returns of the two competing crops. For example, for any return level of $\pi_c$, there will be a critical value $\pi_s^*$ with which continuing in corn-soybean is optimal if $\pi_s < \pi_s^*$ and conversion is optimal if $\pi_s > \pi_s^*$. The $\pi_s^*(\pi_c)$ will form a critical conversion boundary in the $\pi_c - \pi_s$ space. Similarly, there is another conversion boundary from switchgrass to corn-soybean $\pi_{cs}^*(\pi_{sw})$. Following the
standard procedures of solving the real options problems, we can characterize the optimality
conditions of our land conversion decision. In the continuation region (where the agent continues
in current land use), the value functions need to satisfy the following the equation:

\[ rV^i(\pi_c, \pi_s) = \pi_i + \alpha_c \pi_c V^i_{\pi_c} + \alpha_s \pi_s V^i_{\pi_s} + 1/2 \sigma_c^2 \pi_c V^i_{\pi_c} + 1/2 \sigma_s^2 \pi_s V^i_{\pi_s} + \rho \sigma_c \sigma_s \pi_c \pi_s V^i_{\pi_c \pi_s} \]

(4)

This is a no-arbitrage condition expanded by Ito’s lemma, implying that the rate of return of
investing \( V^i \) dollars (left-hand side) should equal the rate of return generated by land use \( i \)
(right-hand side). On the boundaries of conversion, the payoffs of continuing in the current use
should equal the payoffs of converting minus the conversion costs, along with their derivatives.
These are the value-matching and smooth-pasting conditions:

\[ V^i(\pi_c, \pi_s) = V^j(\pi_c, \pi_s) - C_{ij} \quad \text{when } \pi_i = \pi_i^* (\pi_j) \ i, j \in \{c, s\}, \text{ and } i \neq j \]

(5)

\[ \frac{\partial V^i(\pi_c, \pi_s)}{\partial \pi_c} = \frac{\partial V^j(\pi_c, \pi_s)}{\partial \pi_c} \]

and

\[ \frac{\partial V^i(\pi_c, \pi_s)}{\partial \pi_s} = \frac{\partial V^j(\pi_c, \pi_s)}{\partial \pi_s} \quad \text{when } \pi_i = \pi_i^* (\pi_j) \ i, j \in \{c, s\}, \text{ and } i \neq j \]

(6)

The system of equations (4)-(6) subject to (1) implicitly defines the unknown value functions
and two conversion boundaries. Since the land in the alternative use (switchgrass) can be further
converted back to original use (corn-soybean), the option value of converting from corn-soybean
to switchgrass further depends on the option value associated with converting in the other
direction, from switchgrass to corn-soybean. The mutual dependence of the two option values
significantly complicates the solution algorithm. Except in special cases, such as when value
functions are homogeneous of degree one, there is no analytical solution to (4)-(6). Instead, we
solve the model numerically using the collocation method (Miranda and Fackler 2002; Fackler
This method approximates the unknown value functions using a linear combination of \( n \) known basis functions and fixes the coefficients by solving a system of \( n \) equations that are derived from the optimality conditions (4)-(6). Appendix B provides more details.

Table 2.1 presents the parameters we use to solve the model. More details about the parameter estimation are documented in the first essay (Song, Zhao, and Swinton 2010). In summary, historical data on corn and soybean returns were obtained from the U.S. Department of Agriculture (USDA), while data on switchgrass returns were constructed from historical ethanol prices and production cost that are taken from various sources. The drift parameters and variance parameters of the two crop return series were econometrically estimated. The land conversion costs were taken from literature (Khanna, Dhungana, and Clifton-Brown 2008). We assume that the two returns have a correlation coefficient of 0.3, instead of -0.24 as estimated in essay 1.

Historical returns to corn-soybean and switchgrass could be negatively correlated as indicated by our estimation results. Part of the reason is that switchgrass revenue is simulated as a function of petroleum price and thus highly positively correlated with petroleum, whereas until 2005, corn-soybean returns were negatively correlated with petroleum prices due to petroleum used as transportation and fertilizer inputs. However, this pattern of negative correlation could change as more corn and soybean are used to produce biofuels, and as agricultural and petroleum markets become more integrated. Then high petroleum prices may push up corn and soybean prices, increasing their returns. A supporting evidence is that the correlation between the annual ethanol price and corn-soybean return for year 1982-2005 is -0.07, and it changes to 0.28 for year 2006-2008. Tyner (2009) shows similar result that the price correlation between crude oil and corn change from -0.29 during period 1988-2005 to 0.8 during period 2006-2008. Furthermore, the positive correlation may become stronger as switchgrass or other energy crops expand production and compete with corn-soybean for limited land.
Figure 2.1 shows the two boundaries for conversions from corn-soybean to switchgrass ($b^{CS}$) and from switchgrass to corn-soybean ($b^{SC}$). The two boundaries divide the $\pi_c - \pi_s$ space into three regions. Above the boundary $b^{CS}$, it is optimal to convert from corn-soybean to switchgrass. Below the boundary $b^{SC}$, it is optimal to convert from switchgrass to corn-soybean. Between the two boundaries, it is optimal to keep land in its current use. The large inaction zone indicates significant hysteresis in land conversion decisions. For instance, the calculated switchgrass returns based on 2008 prices is $135/acre$ while the corn-soybean return in 2008 is $92/acre$ (both in 1982 dollars).  

If the land is currently in corn-soybean, the minimum switchgrass return for converting the land to switchgrass is $b^{CS}(92) = 295/acre$, which is significantly higher than the $99/acre$ threshold value under ENPV rule.  

Thus, the land will be kept in a corn-soybean rotation even though $\pi_{sw} > \pi_{cs}$. Conversely, if the land is already in switchgrass, the required minimum corn-soybean return for converting into corn-soybean is about $280/acre$. Thus, the land currently in switchgrass will not be converted either.

**Decision under Different Subsidies**

Above we have described various subsidies for supporting energy crop production currently used or proposed in many countries. They can be categorized into four types: (a) a constant annual subsidy, denoted by $f$; (b) a variable annual subsidy, which is a percentage of return, denoted by $\eta$; (c) an insurance policy, which guarantees a minimum annual return of $\pi_s$ from energy crops; (d) a constant annual benefit ($r$).

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17 2009 corn and soybean returns are not available yet from USDA.

18 The conversion boundaries under ENPV are: $b_{NPV}^{CS}(\pi_c) = \pi_c \frac{r - \alpha_s}{r - \alpha_c} + (r - \alpha_s) C_{CS}$ for conversion from corn-soybean to switchgrass, and $b_{NPV}^{SC}(\pi_s) = \pi_s \frac{r - \alpha_c}{r - \alpha_s} - (r - \alpha_c) C_{SC}$ for conversion from switchgrass to corn-soybean.
and (d) a lump-sum payment made to the switchgrass grower either for the first year of growing switchgrass or for the reestablishment after a 10-year rotation, denoted by $s$. The constant subsidy and variable subsidy are abstracted from annual payments used in European countries and the United States, respectively. The insurance subsidy is a mimic of the proposed revenue-based commodity support program in FCEA, which provides payment to farmers when the market revenue falls below an expected or target revenue (more details can be referred to Cooper 2009).

If farmers are risk-neutral and make decisions according to the ENPV rule, different forms of subsidies can give them the same incentive to convert land to energy crops as long as they have the same ENPV by equation (2). This implies that for a given governmental budget, these subsidies will perform the same in terms of attracting the land to grow energy crops. However, using the dynamic land conversion decision model developed above, we will show that ENPV-equivalent subsidies can affect the land conversion costs, instantaneous returns and variability of energy crops differently, causing the optimal land conversion rules to differ.

For each type of subsidy, the value functions need to satisfy the corresponding Bellman equations in the continuation region and the value matching and smooth pasting conditions on the boundaries of conversion. These conditions are summarized in Table 2.2. Constant and variable subsidies will be added to the instantaneous return to switchgrass, which are $\pi_s + f$ and $\pi_s + \eta \pi_s$, respectively. Under an insurance subsidy, the instantaneous payment in Bellman equation of $V^s$ is $\max (\pi_s, \pi_s)$. The value-matching conditions and smooth pasting conditions for these subsidies are the same as (5) and (6). For a one-time cost-sharing subsidy, the Bellman equations for $V^i$ are the same as (4), but the conversion cost $C_{cs}$ is reduced by $s$ in the value-matching condition for converting from corn-soybean to switchgrass. The smooth pasting condition is the same as (6).
The farmer’s optimal land conversion rule under different forms of subsidy will be solved using the same projection method described in Appendix B. The subsidy levels need to be determined before the optimal land conversion model is solved. To make a meaningful comparison, we need to calibrate the subsidy parameters such that the ENPVs of governmental payments over a period are the same. The details about the calibration are presented in the next section.

**Simulation of Land Use Choice**

Given the optimal land conversion rule, a representative corn-soybean grower will convert land to switchgrass when \( b^{cs} \) is reached while a representative switchgrass grower will convert to corn-soybean when \( b^{sc} \) is reached. With stochastic returns, we can compute the *ex ante* expected probability of a unit of corn-soybean land converting to switchgrass within a period of time. Previous real option literature (e.g. Leahy 1993; Pyndick and Dixit 1994) has show that in a competitive industry the optimal investment policy derived in a single-firm partial equilibrium setting happens to coincide with the optimal policy rule in a general equilibrium if all firms share the same risky return process. Given a large number of firms in that industry, the *ex ante* probability of investment will also measure the fraction of available investment we can expect to be implemented. Following Metcalf and Hasett (1995) and Sarkar (2003), we assume that the farmers are homogenous and subject to the same stochastic processes of crop returns. We also assume that governmental has a subsidy program whose goal is to cost effectively attract more land to grow energy crops. Given the same governmental budget, the higher proportion of land that a subsidy program can convert into switchgrass, the more effective it is.

Given the optimal land conversion rules, we can simulate how the farmer responds to the changes of land use returns and calculate the governmental costs under different forms of subsidy.
The simulation steps are illustrated by figure 2.2 and summarized in the following. Note that the steps in dotted rectangular are repeated.

First, we simulate \( N (=5000) \) sample paths of corn-soybean and switchgrass returns over 30 years according to the joint stochastic processes parameterized by values in table 2.1, denoted by 
\[
(\pi_{c,n}(t), \pi_{s,n}(t)) \quad \text{for } n=1,2,...,5000 \text{ and } t=0,1,2,...,30.
\]
This is done with the Econometric Toolbox in Matlab. The initial returns are assumed to be at 2008 level, \( \pi_c(0) = \$92/\text{acre} \) and \( \pi_s(0) = \$135/\text{acre} \), respectively. The initial land use is corn-soybean production.

Second, for each type of subsidy, we initially select a subsidy level and solve the land conversion decision rules.

Third, for each simulated path of corn-soybean and switchgrass returns, given critical land conversion boundaries under different types of subsidies, we can predict the land use assuming that the farmer acts according to the optimal land use decision rule. Each sample path of the two returns, \( \{(\pi_{c,n}(t), \pi_{s,n}(t)), t = 1,...,30\} \forall n \), is compared with the conversion boundaries, 
\[
(b^{cs}(\bullet), b^{sc}(\bullet)),
\]
to decide whether the land is kept in its current crop or should be converted to the alternative crop. For instance, in year 1, when the land is still in corn-soybean, the realized returns on a particular sample path, \( (\pi_{c,n}(t), \pi_{s,n}(t)) \), are compared with boundary \( b^{cs} \). If the realized returns are in the “no action zone” (e.g., if \( \pi_{s,n}(1) \leq b^{cs}(\pi_{c,n}(1)) \) according to the optimal decision rule), the land is kept in corn-soybean, and similar comparisons are made in year 2. If, on the other hand, the realized returns are in the “conversion zone” (i.e., if \( \pi_{s,n}(1) > b^{cs}(\pi_{c,n}(1)) \)), the land is converted to switchgrass, and in year 2, the realized returns \( (\pi_{c,n}(2), \pi_{s,n}(2)) \) will be compared with boundary \( b^{sc} \) to decide whether the land
should be converted into corn-soybean. We can also predict governmental payments based on the farmer’s land use choice. Under constant subsidy, variable subsidy and insurance subsidy, the government pays the farmer \( f / \text{acre} \), \( \eta \pi_s / \text{acre} \) and \( \max(0, (\pi_s - \pi_s)) / \text{acre} \) per year, respectively when the farmer is in switchgrass production. Under the cost-sharing subsidy, once the farmer converts land from corn-soybean to switchgrass\(^{19}\) or reestablishes after ten years of being in switchgrass, the government will pay \( s / \text{acre} \) to the farmer. For each simulated path of corn-soybean and switchgrass returns, we calculate the NPVs of total governmental payments over 30 years for each type of subsidy. Then the means (ENPV) and standard errors of the discounted governmental costs over the N simulated paths of the joint returns can be obtained for each type of subsidy during a 30 year period.

Fourth, we calibrate the subsidies by repeating steps 1-3 so that the ENPVs of governmental costs at the end of 30 years under different subsidies are equalized, at a level of $30/acre (± $1 simulation error). The calibrated subsidy parameters are presented in table 2.3. For each period we count the number of sample paths on which the land is in switchgrass. Dividing this number by N, we obtain the proportion of land in switchgrass for each form of subsidy during a 30 year period.

Results

Critical land conversion boundaries under different forms of subsidies

In this section we present the effects of different subsidies on a representative farmer’s optimal land conversion rule. In figure 2.3a-d, the solid curves are the critical boundaries \( b^{CS} \) and \( b^{SC} \).\(^{19}\)

\(^{19}\) The government can require that a farmer has to stay in switchgrass for some minimum number of years to receive the cost-sharing subsidy; otherwise he has to pay a penalty. In the simulation, we impose that the farmer can receive the subsidy only if he did not convert from switchgrass to corn-soybean in the past five years.
under the no subsidy base case. The dashed curves are conversion boundaries under the four different subsidies.

A constant subsidy increases the instantaneous return to switchgrass. As expected, we can see from figure 2.3a that it lowers the conversion boundary from corn-soybean to switchgrass and raises the conversion boundary from switchgrass to corn-soybean. So it encourages farmers to convert to energy crops and discourages them from withdrawing.

Compared with a constant subsidy, a variable subsidy not only increases the switchgrass return but also its variability. This implies two opposite effects on the optimal land conversion decision: a higher return gives incentive to convert to switchgrass and a disincentive to withdraw land out of it, while more uncertainties will hold back converting to switchgrass and encourage converting out. Figure 2.3b shows that the return effect dominates the uncertainty effect on converting to switchgrass but the uncertainty effect dominates the return effect on converting out so that both $b^{CS}$ and $b^{SC}$ are lowered compared with no subsidy case. However, we should be aware that this is not always the case.

Farmers are assumed to receive the insurance subsidy only when the switchgrass return is lower than $\pi_s$, which is $60/acre$. The insurance subsidy generally lowers the conversion boundary from corn-soybean to switchgrass, but the effect is more dramatic when the corn-soybean return is lower than $25/acre$: farmers will convert to switchgrass even if its market return is zero since the subsidy can increase it to $60/acre$. Similarly, the subsidy raises the conversion boundary from switchgrass to corn-soybean much more when the switchgrass return is lower: for a switchgrass market return lower than $45/acre$ the farmers will not convert to corn-soybean until the latter reaches at least $120/acre$. These effects gradually vanish when the switchgrass return goes beyond the insured level.
Different from the annual subsidies, a cost-sharing subsidy for switchgrass always lowers both direction land conversion boundaries (figure 2.3d). While reducing the conversion costs from corn-soybean to switchgrass ($C_{CS}$) makes the corn-soybean grower less reluctant to covert the land, it also has the indirect effect of making the switchgrass grower more prone to convert back to corn-soybean. This is because although the farmer currently growing switchgrass will not directly benefit from the subsidy for conversion to switchgrass, its existence reduces the expected cost of converting from corn-soybean back to switchgrass, thereby reducing the implied cost of switching to corn-soybean. Thus it indirectly increases his incentive to convert land to corn-soybean. The direct effect of lowering $C_{CS}$ is greater than the indirect effect. The reduction in $C_{CS}$ lowers the boundary from corn-soybean to switchgrass more than the boundary from switchgrass to corn-soybean.

*The Proportion of Land in Switchgrass*

The effects of a subsidy program on encouraging energy crop production can be illustrated more clearly using the proportion of land in switchgrass over a 30 year period. By changing the optimal land conversion decision rule, the subsidy program will change the probability of land converted into energy crops as well as converting out. The lower the conversion boundary from corn-soybean to switchgrass, the more likely the realized returns can reach the boundary, so that the farmer will convert to switchgrass. Conversely, the lower the conversion boundary from switchgrass to corn-soybean, the more likely the realizations of the returns can reach the boundary, so that the farmer will convert out of switchgrass.

Figure 2.4 shows the expected proportion of a unit corn-soybean land converting into switchgrass over a 30 year period under the no subsidy case and the four single subsidy program, given that the expected governmental cost is uniformly $30/acre at the end of 30 years. The
cumulative proportion is not monotonically increasing over the years because the farmer can optimally convert back to corn-soybean when its return is high enough and reach the conversion boundary $b^{SC}$. We first examine the case without subsidy, indicated by the solid curve in figure 2.4. At the beginning, the proportion of land converted into switchgrass increases over years and peaks at 0.26 in year 9. However, the switchgrass return also has a higher level of uncertainty, and eventually land in switchgrass is likely to be converted back to corn-soybean. At the end of the 30 years, the proportion of land in switchgrass is about 0.13. The average probability of land in switchgrass over 30 years is 0.18.

A constant subsidy lowers $b^{CS}$ and raises $b^{SC}$, implying that it is easier to convert into switchgrass and harder to convert out. The conversion pattern over time is similar to the no subsidy case but the proportion of land in switchgrass peaks at 0.28 and stabilizes at 0.19, increased by 0.03 and 0.06 compared with the no subsidy case. The average proportion over all 30 years is also increased from 0.18 to 0.22. In contrast, the variable subsidy and cost-sharing subsidy lower both $b^{CS}$ and $b^{SC}$ (although for different reasons as we discussed above), implying that it is easier to convert into switchgrass as well as to convert out. These two subsidies raise the peak probability of land in switchgrass to 0.28 and 0.32, respectively, but barely change where it stabilizes. The average proportion of land in switchgrass over years is increased from 0.18 to 0.21 under both cases.

When both of the corn-soybean and switchgrass returns are low, the insurance subsidy effectively makes switchgrass the dominant choice. Once the return of corn-soybean falls below $25/acre, the land will be converted to switchgrass and will not be converted out until the corn-soybean return bounces back to at least $105/acre. So the insurance subsidy increases the probability of land in switchgrass the most, peaking at 0.31 and stabilizing at 0.25. The average proportion over all 30 years is 0.2.
The Change and Variation of Governmental Costs over Years

Each subsidy program is calibrated such that it will incur the same expected governmental cost at the end of 30 years. However, these costs may change from year to year. This information is interesting because the government may prefer a policy program that has a stable stream of expenditures. Figure 2.5a shows the mean NPV of governmental costs under each subsidy over 30 years. Since the cost-sharing subsidy is a relative large one-time payment and more conversion to switchgrass happen in the first several years, its expenditure grows faster in the first three years, slows down until intermediate period and stabilizes after year 15. The other three subsidies are annual payments, which increase steadily over the years.

We have assumed the risk-neutrality of government and an ex ante budget constraint and compared the cost-effectiveness of different subsidies. However, the performance of different subsidies could change if the government is risk averse, or has an ex post budget or both. Then less variability of the governmental expenditures will be more desirable. Figure 2.5b shows the simulated standard errors of the NPV of the governmental costs for each subsidy program over the 30 years. The standard error of the cost-sharing subsidy payment rises rapidly in the first three years and becomes steady at $36/acre since then. The standard error of the constant subsidy payment keeps rising steadily to 44/acre at the end of 30 years. The distributions of other two subsidy payments are much more heavy-tailed. The simulated standard error is $143/acre and $72/acre, or 4.8 and 2.4 times of the mean under the variable subsidy payment and the insurance subsidy payment, respectively, at the end of 30 years.

The Effects of Combining Cost-sharing Subsidy with Other Types of Subsidies

In addition to considering the program implementing a single subsidy, we also evaluate the effectiveness of combining the lump-sum cost-sharing subsidy with other three types of annual subsidy, as often occurs in practice. For example, as we discussed above, Irish farmers can
receive a subsidy up to half of establishment costs as well as a constant subsidy of $70/acre for planting willow and miscanthus, while U.S. farmers can receive a subsidy up to 75% of the establishment costs and a 2-year variable subsidy that matches the biorefiner’s payment for any eligible energy crop.

Again we compare the three forms of combined subsidies among themselves and with their single form subsidy counterpart by how they change the expected proportion of a unit of corn-soybean land converting to switchgrass. The simulation is performed given an expected governmental cost at $80/acre. The calibrated subsidy levels are presented in table 2.4. First we can examine the relative performance of the three combined subsidies. Figures 2.6 a-c show that consistent with the relative performance of the single subsidy forms, subsidizing the establishment costs and insuring a minimum return will result in the highest probability of land in switchgrass, which peaks at 0.4 and stabilizes at 0.3 at the end of 30 years and averages at 0.33. A constant subsidy together with a cost-sharing subsidy has 0.4 probability of land in switchgrass at the peak and 0.22 at the end of 30 years, averaging at 0.29. A variable subsidy together with a cost-sharing subsidy will rank lowest, having probability of 0.3 for land growing switchgrass at peak and 0.13 at the end of 30 years and averaging 0.21.

Compared with their single subsidy counterpart, the combined forms increase the peaking proportion of land in switchgrass in the intermediate period but reduce the proportion toward the latter part of a 30 year time horizon (except the variable subsidy together with cost-sharing subsidy, whose proportion of land in switchgrass is slight higher than its counterpart). This can be explained by the dual effects of the cost-sharing subsidy on land conversion: it has a positive effect on the expected rate of converting land to switchgrass by lowering the conversion boundary $b^{CS}$ more than the annual subsidies but also has a negative effect by inducing land converting out later since it lowers the conversion boundary $b^{SC}$. The average probability over years change little compared to the single forms, but single form subsidies have smaller variances.
Conclusions and Discussions

This study examines the design of agricultural subsidy programs that aim to encourage desirable land use using a real options framework that reflects the following features: (a) the dynamic characteristics of land conversion; (b) the sunk costs and future return uncertainties associated with land conversion; and (c) flexibility in an optimizing, representative farmer’s land use decisions. Results show that failure to consider these factors can lead to misleading conclusions. Although the levels of different subsidy forms were selected to be ENPV-equivalent, they are not equally cost-effective.

Using energy crop production as an example, we compare three annual subsidies and one lump-sum subsidy that have the same expected governmental costs. The insurance subsidy results in the highest expected proportion of land being converted to energy crops (switchgrass), followed by the constant subsidy. Although the cost-sharing subsidy and the variable subsidy have the positive effect of encouraging land conversion to switchgrass, they also have a negative effect of discouraging land from staying in that switchgrass. The two effects cancel each other out and result in an increase in the predicted proportion of land in switchgrass in the intermediate period but a drop back to the no-subsidy level at the end of 30 years. The relative performance of combining cost-sharing subsidy with other annual subsidies is consistent with comparison of single subsidies.

The results presented in this paper suggest that the existing U.S. energy crop subsidy system, which is a variable subsidy combined with a cost-sharing subsidy, may not be the most cost-effective. Greater cost-effectiveness of the insurance subsidy highlights the research needs for how to reduce the uncertainties of the returns to energy crops. Taheripour and Tyner (2008) propose a subsidy that is inversely related with the oil price in order to reduce the volatility of

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20 They call it the variable subsidy, which clearly is different from the one in our paper.
energy crop prices. Compared with the government providing an insurance policy, the long-term contract between energy crop growers and biorefiners may serve as a better mechanism considering the possible transaction costs involved in the former.

There is a caveat in evaluating the performance of cost-sharing subsidy based on our results. We only consider the cost-effectiveness of a subsidy, i.e., the ability to convert land to switchgrass given the same governmental expenditures. But there are other factors that justify the cost-sharing subsidy, one of which is the farmer’s liquidity constraint. Numerous studies show farmers are concerned about the large up-front costs of establishing the energy crops (e.g. Sherrington, Bartley and Moran 2008; Bocqueho and Jacquet 2010). A cost-sharing subsidy can relax this constraint and thus reduce the adoption barriers.
Table 2.1. Parameters for Solving Optimal Land Conversion Rule without Subsidies

<table>
<thead>
<tr>
<th>Land Conversion Model Parameters</th>
<th>Notation and Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$r$ 0.08</td>
</tr>
<tr>
<td>Drift Rate of Corn-soy Return</td>
<td>$\alpha_c$ 0.03</td>
</tr>
<tr>
<td>Drift Rate of Switchgrass Return</td>
<td>$\alpha_s$ 0.03</td>
</tr>
<tr>
<td>Variance Parameter of Corn-soy Return</td>
<td>$\sigma_c$ 0.29</td>
</tr>
<tr>
<td>Variance Parameter of Switchgrass Return</td>
<td>$\sigma_s$ 0.64</td>
</tr>
<tr>
<td>Correlation Coefficient between Two Returns</td>
<td>$\rho$ 0.3</td>
</tr>
<tr>
<td>Land Conversion Cost from Corn-soy to Switchgrass</td>
<td>$C_{CS}$ $136/acre$</td>
</tr>
<tr>
<td>Land Conversion Cost from Switchgrass to Corn-soy</td>
<td>$C_{SC}$ $47/acre$</td>
</tr>
</tbody>
</table>
Table 2.2 Land Conversion Optimality Conditions under Different Subsidies

<table>
<thead>
<tr>
<th>Subsidy Type</th>
<th>Bellman Equations</th>
<th>Value Matching Conditions</th>
<th>Smooth Pasting Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost-sharing</td>
<td>Same as (4)</td>
<td>Same as (5) for ( i = c ); ( V^c(\pi_c, \pi_s) = V^s(\pi_c, \pi_s) - (C_{cs} - s) ) when ( \pi_s = b^{cs}(\pi_c) ) for ( i = s )</td>
<td>Same as (6)</td>
</tr>
<tr>
<td>Constant</td>
<td>( rV^S(\pi_c, \pi_s) = \pi_s + f + \alpha_c V^S \pi_c + \alpha_s V^S \pi_s )</td>
<td>Same as (5)</td>
<td>Same as (6)</td>
</tr>
<tr>
<td></td>
<td>(+1/2\sigma_c^2 V^S \pi_c^2 + 1/2\sigma_s^2 V^S \pi_s^2 + \rho \sigma_c \sigma_s \pi_c \pi_s V^S \pi_c \pi_s )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>Same as (4) for ( i = c ); for ( i = s )</td>
<td>Same as (5)</td>
<td>Same as (6)</td>
</tr>
<tr>
<td></td>
<td>( rV^S(\pi_c, \pi_s) = (1+\eta)\pi_s + \alpha_c V^S \pi_c + \alpha_s V^S \pi_s )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(+1/2\sigma_c^2 V^S \pi_c^2 + 1/2\sigma_s^2 V^S \pi_s^2 + \rho \sigma_c \sigma_s \pi_c \pi_s V^S \pi_c \pi_s )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Insurance</td>
<td>Same as (4) for ( i = c ) for ( i = s )</td>
<td>Same as (5)</td>
<td>Same as (6)</td>
</tr>
<tr>
<td></td>
<td>( rV^S(\pi_c, \pi_s) = \max(\pi_s, \pi_s) + \alpha_c V^S \pi_c + \alpha_s V^S \pi_s )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(+1/2\sigma_c^2 V^S \pi_c^2 + 1/2\sigma_s^2 V^S \pi_s^2 + \rho \sigma_c \sigma_s \pi_c \pi_s V^S \pi_c \pi_s )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 2.3 Parameters for Policy Simulation: Single Subsidy

<table>
<thead>
<tr>
<th>Subsidy Form</th>
<th>Subsidy Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Subsidy</td>
<td>$12/acre</td>
</tr>
<tr>
<td>Variable Subsidy</td>
<td>2%</td>
</tr>
<tr>
<td>Insurance Subsidy</td>
<td>$60/acre</td>
</tr>
<tr>
<td>Cost-sharing Subsidy</td>
<td>$90/acre</td>
</tr>
<tr>
<td><strong>Governmental Costs</strong></td>
<td><strong>$30/acre</strong></td>
</tr>
</tbody>
</table>

Note: The simulation errors of expected governmental costs across different subsidies are within $1/acre.
## Table 2.4 Parameters for Policy Simulation: Single Subsidy vs. Combined Subsidy

<table>
<thead>
<tr>
<th></th>
<th>Combined Form</th>
<th>Single Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost-sharing Subsidy</td>
<td>68/acre (Half of Baseline Conversion Costs)</td>
<td></td>
</tr>
<tr>
<td>Constant Subsidy</td>
<td>$14/acre</td>
<td>$24/acre</td>
</tr>
<tr>
<td>Variable Subsidy</td>
<td>3.6%</td>
<td>5.5%</td>
</tr>
<tr>
<td>Insurance Subsidy</td>
<td>$61/acre</td>
<td>$85/acre</td>
</tr>
<tr>
<td>Governmental Costs</td>
<td>$80/acre</td>
<td></td>
</tr>
</tbody>
</table>

Note: The simulation errors of expected governmental costs across different subsidies are within $2/acre.
Figure 2.1 Optimal land conversion rule: no subsidy
1. Simulate the realized returns of corn-soybean and switchgrass for 5000 times using known parameters of the stochastic processes and 2008 return values over 30 years.

Obtain 5000 paths of return pairs over 30 years.

2. Select a subsidy level and solve the land conversion model.

Obtain critical land conversion boundaries under each type of subsidy.

3. Predict the land use in each period and the governmental costs.

Outputs are state of land use, the means and standard errors of expected governmental costs in each period.

4. Repeat steps 1-3 for each type of subsidy until their expected governmental costs at end of 30 years are equal.

Obtain the proportion of land in switchgrass, means and standard errors of governmental costs over the 30 years.

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Figure 2.2 Land conversion simulation steps
Figure 2.3a Optimal land conversion rule: constant subsidy
Figure 2.3b Optimal land conversion rule: variable subsidy
Figure 2.3c Optimal land conversion rule: insurance subsidy
Figure 2.3d Optimal land conversion rule: cost-sharing subsidy

Figure 2.4 Probability of land in switchgrass: comparison of single subsidy (expected NPV of governmental costs=$30/acre)
Note: The average proportion of land in switchgrass over 30 years: 0.25 for the insurance subsidy, 0.22 for the constant subsidy, 0.21 for the cost-sharing subsidy, 0.20 for the variable subsidy, and 0.18 for the no subsidy case.
Figure 2.5a Mean NPVs of governmental costs over years
Figure 2.5b Standard errors of NPV of governmental costs over years
Figure 2.6a  Effect of combining a cost-sharing subsidy with an insurance subsidy (expected NPV of governmental costs is $80/acre)
Figure 2.6b Effect of combining a cost-sharing subsidy with a constant subsidy (expected NPV of governmental costs is $80/acre)
Figure 2.6c Effect of combining cost-sharing subsidy with a variable subsidy (expected NPV of governmental costs is $80/acre)
References


Concluding Remarks

Large scale production of dedicated energy crops could have dramatic effects on land use change and associated economic and environmental impacts. However, whether farmers are willing to adopt these crops is still an open question. This dissertation, which consists of two essays, looks at a representative farmer’s optimal land use decisions for energy crops and the cost-effectiveness of various governmental subsidies that occur in practice.

This dissertation applies a new real options framework to model a farmer’s land conversion decision, which eliminates the absolute irreversibility assumption and allows costly reversion of land use. By comparison with the real options results, an NPV model predicts that an optimizing farmer would be much more prone to convert land to energy crops. A one-way real option model characterizing the land conversion decision as irreversible predicts much greater reluctance to convert land from traditional crops to energy crops.

The results from the first essay suggest that sunk costs and risk tend to deter farmers’ willingness to invest in dedicated energy crops, causing them to require more than double the estimated break-even revenue based on average yields and prices that ignore variability. Using rotated corn-soybean and switchgrass as two representative crops, the simulation shows that the predicted probability of land converting from corn-soybean to switchgrass over a 30 year period is low. Governmental subsidies can increase the predicted probability, as shown in essay 2. Subsidies that reduce the uncertainties of return to energy crop perform better than the ones that increase the uncertainties. However, the effectiveness of cost-sharing subsidy is not as good as expected because it
has a counter-intuitive side-effect of encouraging land converted out of the energy crop at a later period. The results presented in this paper suggest that the existing U.S. energy crop subsidy system, which is a variable subsidy combined with a cost-sharing subsidy, may not be the most cost-effective. Greater cost-effectiveness of the insurance subsidy highlights the research needs for how to reduce the uncertainties of the returns to energy crops.

This study can be extended in several aspects. First, the representative farmer is assumed to be risk-neutral. To maximize the NPV of current and future returns, it is optimal to convert all his land to the alternative use if the return of alternative land use is high enough compared to the return of current land use. Therefore, crop diversification is never optimal. If the risk-aversion is introduced, the crop diversification may become optimal since it is often used as a risk reduction strategy. The model will be modified to maximize an expected utility function that includes the risk-aversion. The control variable could be the proportional land in each use. This modification may further complicate the solution algorithm.

Second, our optimal land conversion decision is derived in a representative farmer partial-equilibrium setting, in which the individual’s ex ante probability of land conversion can be simulated over a certain period. Although the predicted ex ante probability could measure the fraction of land in switchgrass under the homogeneity assumption, two more factors should be accounted for a better measurement on the aggregate level. One is that reallocation of land into different crops may have feedback effects, which could make the stochastic processes of the crop returns endogenous. Another is to model the farmers’ heterogeneity. Incorporating these features may need a
general stochastic dynamic model. There are works that employ the computational general equilibrium models or mathematical programming models to predict the land use change on a regional or national level (e.g. Keeney and Hertel 2009; Walsh et. al. 2003). However, most of these models fail to account for uncertainties. How to integrate the micro-level model that account for the adjustment costs and the uncertainties in the land conversion with the macro-level model that account for the endogenous evolution of crop returns and farmers’ heterogeneities is an interesting topic.
References
